

Name: _____

Discussion Section: _____

Solutions should show all of your work, not just a single final answer.

5.5: The Substitution Rule

1. Evaluate each of the following indefinite integrals using substitution, expressing your final answer in terms of x .

(a) $\int x^2 \sin(x^3) dx$

(b) $\int x\sqrt{4x+1} dx$

(c) $\int \frac{x}{x^2+1} dx$

(d) $\int \frac{1}{x \ln x} dx$

2. Rewrite each of the following definite integrals in x as a definite integral in the indicated new variable u . **Do not evaluate** the new definite integral.

(a) $\int_0^1 x^2(1 + 2x^3)^5 dx$ in terms of $u = 1 + 2x^3$

(b) $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x} dx$ in terms of $u = \cos x$

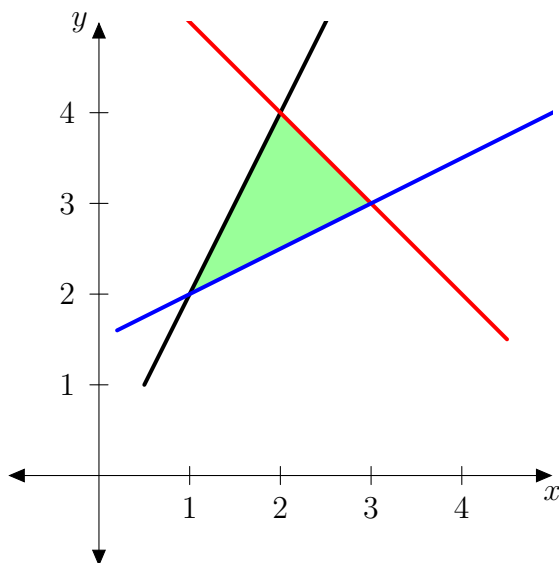
(c) $\int_0^{\pi/3} \sin x \cos x dx$ in terms of $u = \cos x$

(d) $\int_2^3 xe^{-x^2} dx$ in terms of $u = x^2$

3. (T/F) When $u = \sqrt{x}$, $\int_0^4 f(\sqrt{x}) dx = \int_0^2 2uf(u) du$.

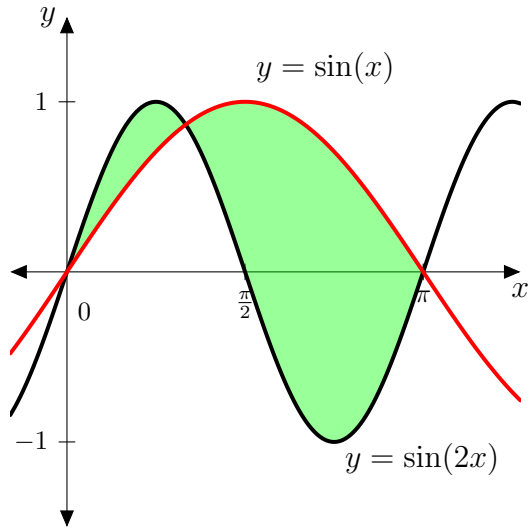
6.1: Areas Between Curves

4. We want to find the area of the triangle with vertices $(1, 2)$, $(2, 4)$, and $(3, 3)$, by calculus.

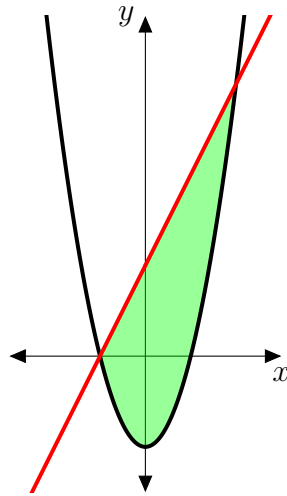


- (a) Find equations for the three lines connecting the vertices.
- (b) Use the equations in part (a) to express the area of the triangle in terms of integrals, and then compute the area.

5. Find the area of the regions below, between $y = \sin x$ and $y = \sin(2x)$ for $0 \leq x \leq \pi$. (Hint: To find the exact coordinates of the point where the graphs cross, recall that $\sin(2x) = 2 \sin x \cos x$.)



6. We want to find the area of the region bounded by $y = 2x + 4$ and $y = x^2 - 4$.



(a) Determine the coordinates of the points where the line and parabola intersect.

(b) Express the area as an integral with respect to x .

(c) Express the area as an integral with respect to y .

(d) Explain which of (a) or (b) is simpler to compute, and use the simpler one to find the area. Simplify your final answer.