Math 1131 Week 10 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

4.7: Optimization Problems

1. A closed box (top, bottom, and all four sides) needs to be constructed to have a volume of $9\,\mathrm{m}^3$ and a base whose width is twice its length. See Figure 1.

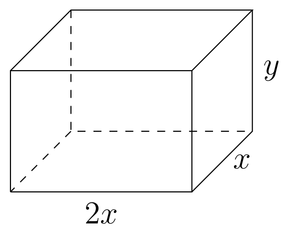

Figure 1: A box

Use calculus to determine the dimensions (length, width, height) of such a box that uses the least amount of material. Justify why your answer corresponds to a minimum, not a maximum.

2. We want to find the points on $y = x^2$ that are closest to $(0, 3)$.

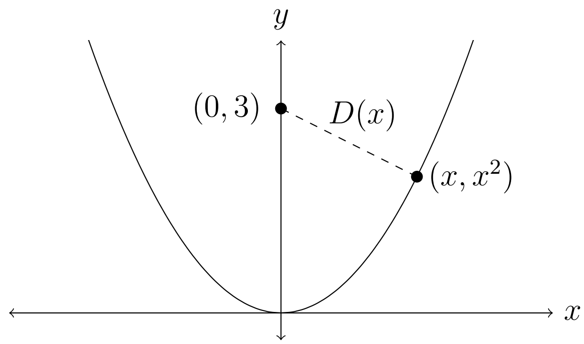

Figure 2: Distance to $(0, 3)$ on $y = x^2$.

(a) For each point (x, x^2) on the parabola, find a formula for its distance to $(0, 3)$. Call this distance $D(x)$. (See Figure 2.)

(b) Let $f(x) = D(x)^2$, which is the *squared distance* between (x, x^2) and $(0, 3)$. Finding where $D(x)$ is minimal is the same as finding where $f(x)$ is minimal. Determine all x where $f(x)$ has an absolute minimum. The points (x, x^2) for such x are the closest points to $(0, 3)$ on $y = x^2$.

3. Three line segments of length 1 are joined together at endpoints to form a base and the legs of an isosceles trapezoid, as in Figure 3. Let θ in $(0, \pi/2)$ be the common angle measurement between the legs and the line passing through the base of length 1. We want to find the angle θ that maximizes the area of the trapezoid.

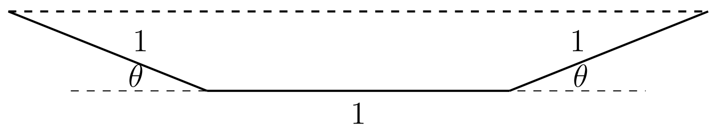

Figure 3: An isosceles trapezoid with base and legs of length 1.

(a) Compute the area $A(\theta)$ of the trapezoid. The general area formula for a trapezoid is $\frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the lengths of the bases. (Hint: Break up the trapezoid into a rectangle with two right triangles at both ends. Use trigonometry to compute the height and the length of the longer base in terms of θ.)

(b) Find all solutions to $A'(\theta) = 0$ with $0 < \theta < \pi/2$. (The answer is *not* $\pi/4 = 45^o$.)

(c) Verify that the area $A(\theta)$ is a maximum, not a minimum, at the angle found in part (b) and compute this maximum area.

4.9: Antiderivatives

4. Find the most general antiderivative of the function (use C as any constant).

   (a) $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$

   (b) $f(x) = \frac{10}{x^9}$ for $x > 0$

   (c) $f(x) = \frac{x^4 + 3\sqrt{x}}{x^2}$ for $x > 0$

   (d) $f(x) = \cos x - 5\sin x + e^x$

   (e) $f(x) = e^2$

   (f) $f(x) = 7x^{2/5} + 8x^{-4/5}$ for $x > 0$

5. Find a function $f(x)$ satisfying the given conditions.

(a) $f'''(x) = \cos x,\ f(0) = 1,\ f'(0) = 2$, and $f''(0) = 3$

(b) $f''(x) = 2 - 12x,\ f(0) = 9,\ f(2) = 7$

6. A particle moves along a line according to the following information about its position $s(t)$, velocity $v(t)$, and acceleration $a(t)$. Find the particle's position function $s(t)$ for general t.

   (a) $v(t) = 1.5t^2 + 4t$, $s(4) = 50$

   (b) $a(t) = 3\cos t - 2\sin t$, $s(0) = 0$, $v(0) = 4$

7. T/F (with justification) The antiderivative of $\cos(x^2)$ is $\sin(x^2) + C$.