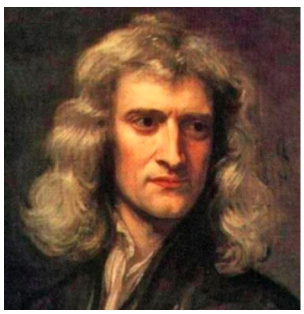
Math 1131: Calculus Overview — Fall 2019

Calculus was created by Isaac Newton and Gottfried Leibniz in the 1600s. It has two parts, differential and integral calculus, which together may be called infinitesimal calculus (or "*the* Calculus" if you want to sound pretentious). The word "calculus" really means "system of calculation" (from the Latin word for "small stone") and in math there are many types of calculus: functional calculus, residue calculus, variational calculus, and so on. The type of calculus due to Newton and Leibniz has so many applications that the undecorated term "calculus" refers to that.

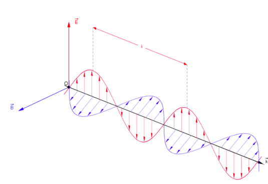


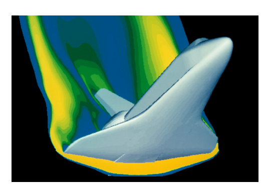
Newton

Leibniz

Newton and Leibniz worked on calculus largely independently (with different motivations, in physics and in geometry), but how much each knew about what the other was doing became a long-running controversy. Some aspects of calculus were known before Newton and Leibniz (ideas in integral calculus can be traced back to ancient Greece), but we give credit to Newton and Leibniz because they were the first to realize the generality of calculus to solve problems.

What makes calculus different from earlier topics in math is its relentless use of *infinity*: the infinitely small and the infinitely large. Calculus lets us break problems into infinitely many small parts, solve those, and then put them back together. What we can do with calculus that we can't do with algebra alone is *model dynamic change*. *Without* calculus, we can study cars at a constant velocity, circuits with a constant current, or constant fluid flow (the fluid could be a liquid or gas, like air). *With* calculus we can study varying velocity, varying current, and varying fluid flow.





A TED talk by Jeff Heys here gives examples of real world mathematical models that use calculus. Non-invasive medical procedures such as MRI depend on calculus (and physics and engineering). In a book listing what people thought was the greatest invention in the last 2000 years, there are many answers (Internet, universal education, *etc.*) and Bart Kosko's reply is calculus. He wrote "The world today would be very different if the Greeks and not Newton/Leibniz had invented or discovered calculus. The world today might have occurred a millennium or two earlier."

While calculus is the math of dynamic change, it helps solve problems that at first may not appear to be about anything dynamic. An example is optimization problems: find the input making a given function as large or small as possible. The way calculus gets involved is by *thinking* about the problem in a dynamic way, using calculus on that, and then (sometimes) forgetting how calculus got used if the end of the solution is algebra. Optimization problems solved with calculus occur in many areas, such as economics (constrained optimization), machine learning (gradient descent), and statistics (least squares).[1]

Remark. Thanks to fast computers, calculus-based formulas in solutions to problems are often hidden in computer code, so people who use calculus-based algorithms in software on a computer may have no idea how the algorithms work, or may even think no fancy math is involved. Someone who wants to improve the algorithm, however, would need to understand it. An analogy: you can be good at driving a car without knowing how a car works, but this doesn't mean you can become an automotive engineer without knowing any math beyond algebra.

A few short essays on the most basic ideas of calculus written by applied math professor Steven Strogatz are here, here, and here, and he wrote a book about the societal impact of calculus (supporting and supported by developments in science and technology). A video series on the "essence of calculus" created by YouTube math educator Grant Sanderson, a.k.a. 3Blue1Brown, is here. Strogatz's first essay and Sanderson's first video present different ways to explain why the area of a circle is πr^2, well worth comparing. Some topics covered by Sanderson (*e.g.*, Taylor series near the end) are taught in second-semester calculus, so if you look over all of his calculus videos now, plan to go back and watch them again as you learn more.

[1] The mathematical description of these optimization applications needs multivariable calculus, which is beyond the scope of this course. But you can't get there without a solid understanding of this course first.