Curl and Divergence

1. For each of the following, either compute the expression or explain why it doesn't make sense. Assume that $f(x, y, z) = x^2y + xz - 1$ and $\vec{F} = \langle z, x, y \rangle$.

   (a) $\text{div}(\text{curl} f)$

   (b) $\text{curl}(\text{div}\vec{F})$

   (c) $\vec{\nabla} f \cdot \vec{F}$

   (d) $\text{curl}\vec{\nabla} f$

   (e) $\text{curl}\vec{F} + \text{div}\vec{F}$

   (f) $\text{div}(\vec{\nabla} f + \vec{F})$

2. Using the same scalar function $f(x, y, z)$ and vector field $\vec{F} = \langle z, x, y \rangle$ given above, evaluate each of the following expressions. You may use the Fundamental Theorem for Line Integrals or Green's Theorem if they apply.

   (a) $\int_{C_1} \text{curl}\vec{F} \cdot d\vec{r}$, where C_1 is the line segment from $(-1, 3, 5)$ to $(3, -1, -2)$

   (b) $\int_{C_2} \vec{\nabla} f \cdot d\vec{r}$, where C_2 is the portion of the parabola given by $\vec{r}(t) = \langle t^2, t, 3t \rangle$ with $-1 \leqslant t \leqslant 1$

   (c) $\int_{C_3} \text{div}\vec{F}\, ds$, where C_3 is the curve given by $\vec{r}(t) = \langle e^{t^2}, \ln(t^3 + 1), 1 \rangle$ with $0 \leqslant t \leqslant 5$

   (d) $\int_{C_4} \text{curl}\vec{F} \cdot d\vec{r}$, where C_4 is the ellipse given by $\vec{r}(t) = \langle \cos t, \sin t, \cos t \rangle$ with $0 \leqslant t \leqslant 2\pi$

Answers

1. (a) Doesn't make sense
   (b) Doesn't make sense
   (c) $z(2xy + z) + x^3 + xy$
   (d) $\vec{0}$
   (e) Doesn't make sense
   (f) $2y$
2. (a) -7
   (b) 8
   (c) 0
   (d) 0