Line Integrals vs. Surface Integrals

1. Let $\vec{F} = \langle e^{yz}, xze^{yz}, xye^{yz} \rangle$.
   (a) Show that $\vec{F}$ is a conservative vector field.
   (b) If C is the curve of intersection between the cylinder $x^2 + y^2 = 4$ and the plane $3x - 2y + 7z = 12$ (what is the shape of this intersection?), evaluate the line integral
   $$\int_C \vec{F} \cdot d\vec{r}.$$
   (c) If S is the portion of the xz-plane with $-1 \leqslant x \leqslant 1$ and $0 \leqslant z \leqslant 1$ oriented in the direction of the y-axis, evaluate the surface integral
   $$\iint_S \vec{F} \cdot d\vec{S}.$$
   (d) If S is the surface $z = \sqrt{x^2 + y^2}$ with $0 \leqslant z \leqslant 4$ and outward orientation, evaluate the surface integral
   $$\iint_S \text{curl}\vec{F} \cdot d\vec{S}.$$
2. Let $\vec{F} = \langle -y, z, -x \rangle$.
   (a) If C is the circle $x^2 + y^2 = 4$, $z = 0$, traversed once counterclockwise, evaluate the line integral
   $$\oint_C \vec{F} \cdot d\vec{r}.$$
   (b) If S is the surface $x^2 + y^2 \leqslant 4$, $z = 0$, with upward orientation, evaluate the surface integral
   $$\iint_S \vec{F} \cdot d\vec{S}.$$
   (c) Compute $\text{curl}\vec{F}$.
   (d) If S is the portion of the surface $z = 4 - x^2 - y^2$ with $z \geqslant 0$ and upward orientation, evaluate the surface integral
   $$\iint_S \text{curl}\vec{F} \cdot d\vec{S}.$$

Answers

1. (a) $\text{curl}\vec{F} = \vec{0}$
   (b) 0
   (c) 0
   (d) 0
2. (a) 4π
   (b) 0
   (c) $\langle -1, 1, 1 \rangle$
   (d) 4π