Using the Divergence Theorem

1. Let $\vec{F} = \langle z, y, x \rangle$ and let S be the surface $x^2 + y^2 + z^2 = 16$ with outward orientation.

   (a) Compute the flux of $\vec{F}$ across the surface S using the **definition** of a surface integral. You can use the parametrization $\vec{r}(u, v) = \langle 4 \sin v \cos u, 4 \sin v \sin u, 4 \cos v \rangle$, but make sure you understand where this comes from and could produce it yourself if necessary. Also free to use the following:

   $$\vec{r}_u \times \vec{r}_v = -\langle 16 \sin^2 v \cos u, 16 \sin^2 v \sin u, 16 \sin v \cos v \rangle.$$

   (b) Use the Divergence Theorem to find the flux, and make sure your answer agrees with part (a).

2. Let S be the surface of the solid bounded by $y^2 + z^2 = 1$, $x = -1$, and $x = 2$ and let $\vec{F} = \langle 3xy^2, xe^z, z^3 \rangle$. Calculate the flux of $\vec{F}$ across the surface S, assuming it has positive orientation.

3. Let S be the surface $x^2 + y^2 + z^2 = 4$ with positive orientation and let $\vec{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3 \rangle$. Evaluate the surface integral

   $$\iint_S \vec{F} \cdot d\vec{S}.$$

4. Let $\vec{F} = \langle P, Q, R \rangle$ be a vector field and S a closed surface with positive orientation containing a solid E. Use the Divergence Theorem to compute

   $$\iint_S \text{curl}\vec{F} \cdot d\vec{S}.$$

5. Compute $\iint_S \vec{F} \cdot d\vec{S}$ if $\vec{F} = \langle x^2 y, -xy^2, 4z(z^2 - 1) \rangle$ and S is the unit cube with outward orientation with vertices at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(1, 1, 0)$, $(0, 0, 1)$, $(1, 0, 1)$, $(0, 1, 1)$, and $(1, 1, 1)$, minus the top face (so S is like a box with no lid). Notice that S is not closed. How might we still be able to make use of the Divergence Theorem?

Answers

1. $\frac{256\pi}{3}$
2. $\frac{9\pi}{2}$
3. $\frac{384\pi}{5}$
4. 0
5. 0