Vector Functions and Parameterized Curves

1. Give a vector function for each of the following curves, including the appropriate range of t-values.
   (a) A circle in the plane $x = 4$ centered at $(4, 0, 0)$ with radius 3, traced once (this affects the range of t-values).
   (b) The portion of the curve in the xy-plane with equation $y = \sqrt{x + 1}$ from $(0, 1, 0)$ to $(3, 2, 0)$.
   (c) The portion of the curve in the plane $y = -1$ with equation $x = z^2 - z + 2$ from $(2, -1, 1)$ to $(8, -1, 3)$.

2. Consider the curve given by the vector function $\vec{r}(t) = \langle t^2, 1 - 3t, 1 + t^3 \rangle$.
   (a) Find the value(s) of t for which the given curve passes through the points $(1, 4, 0)$ and $(9, -8, 28)$.
   (b) Show that the curve does not pass through the point $(4, 7, -6)$.

3. We say that two curves $\vec{r}_1(t)$ and $\vec{r}_2(t)$ *intersect* if they ever pass through the same point (this could be at different times) and that they *collide* if they cross at the same time.

   For example, the curves $\vec{r}_1(t) = \langle t - 1, 0 \rangle$ and $\vec{r}_2(t) = \langle \cos t, \sin t \rangle$ intersect at points $(-1, 0)$ and $(1, 0)$, but they do not collide. Before continuing, think about why that is true.

   Say that two missiles are fired with trajectories given by the vector functions

   $$\vec{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle \qquad \text{and} \qquad \vec{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle.$$

   Assuming $t \geqslant 0$, will the missiles collide?

   Hint: It may be helpful to rename the variable in the second trajectory as s, namely $\vec{r}_2(s) = \langle 4s - 3, s^2, 5s - 6 \rangle$. The paths of the missiles will intersect if you can find a pair (t, s) so that each curve passes through the same point, and the missiles will collide if that point is reached when $t = s$.

4. Find an equation for the tangent line to the curve $\vec{r}(t) = \langle e^t, te^t, te^{t^2} \rangle$ at the point $(1, 0, 0)$.

5. A bee flies along the path $\vec{r}(t) = \langle 12t, 8t^{3/2}, 3t^2 \rangle$.
   (a) Find the displacement of the bee from $t = 0$ to $t = 1$.
   (b) Find the distance traveled by the bee from $t = 0$ to $t = 1$.

6. We don't know the equation that defines a certain surface S, but we are able to determine the equation of two curves that lie in the surface and that intersect at the point $(2, 1, 3)$, namely

   $$\vec{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle \qquad \text{and} \qquad \vec{r}_2(t) = \langle 1 + t^2, 2t^3 - 1, 2t + 1 \rangle.$$

   Determine an equation for the tangent plane at the point $(2, 1, 3)$.

Answers

1. (a) $\vec{r} = \langle 4, 3\cos t, 3\sin t\rangle$, $0 \leqslant t \leqslant 2\pi$
   (b) $\vec{r} = \langle t, \sqrt{t+1}, 0\rangle$, $0 \leqslant t \leqslant 3$
   (c) $\vec{r} = \langle t^2 - t + 2, -1, t\rangle$, $1 \leqslant t \leqslant 3$
2. (a) $(1, 4, 0) \to t = -1$, $(9, -8, 28) \to t = 3$
3. Yes
4. One possible answer is $\vec{L}(s) = \langle 1 + s, s, s\rangle$ (using s as the parameter since the curve uses t)
5. (a) $\langle 12, 8, 3\rangle$ (or, as a scalar, $\sqrt{144 + 64 + 9}$)
   (b) 15
6. One possible answer is $24(x - 2) - 14(y - 1) + 18(z - 3) = 0$.