Tangent Planes and Directional Derivatives

1. Find an equation of the tangent plane for $z = x\sin(x+y)$ at $(-1, 1)$.

2. Consider the function $f(x, y) = \dfrac{2x+3}{4y+1}$.
   (a) Find an equation of the tangent plane to the surface $z = f(x, y)$ at $(0, 0)$.
   (b) Use your equation from part (a) to approximate the value of $f(0.01, 0.01)$, and find the actual value of $f(0.01, 0.01)$ rounded to three decimal places to compare.

3. We don't know the equation that defines a certain surface S, but we are able to determine two tangent vectors at the point $(2, 1, 3)$ on the surface, namely

$$\vec{v}_1 = \langle 3, 0, -4 \rangle \qquad \text{and} \qquad \vec{v}_2 = \langle 1, 6, 2 \rangle.$$

   Using this information, determine an equation for the tangent plane at $(2, 1, 3)$.

4. The function $A(x, y) = 4000 + 3xy - 4x^2 - 5y^2$ gives the altitude in feet at any point (x, y) on a hill (we can think of the (x, y) coordinates as essentially giving longitude and latitude). We are currently located on the hill at $(-1, 2)$.
   (a) Find our current altitude.
   (b) Find the initial slope if we start moving in the direction of the vector $\langle 1, 7 \rangle$
   (c) Find a nonzero vector that points in a direction in which the initial slope will be 0 (your vector does not have to be a unit vector).

5. Let $f(x, y) = 8e^{y\sqrt{x}} - x^2y^3$.
   (a) Find the derivative of f in the direction $\langle 5, 3 \rangle$ at the point $(4, -1)$.
   (b) At the point $(4, -1)$, find the direction in which the maximum derivative of f occurs, and find the maximum derivative of f.

Answers

1. $z = -x - y$
2. (a) $z = 2x - 12y + 3$

   (b) Tangent plane approximation 2.9, actual value of 2.904 (rounded to three decimal places)
3. $24(x-2) - 10(y-1) + 18(z-3) = 0$
4. (a) 3,970 feet

   (b) $\dfrac{-147}{\sqrt{50}}$

   (c) Any constant multiple of $\langle 23, 14 \rangle$ will work
5. (a) $\dfrac{38e^{-2} - 104}{\sqrt{34}}$

   (b) Direction is $\langle -2e^{-2} + 8, 16e^{-2} - 48 \rangle$, maximum value is $\sqrt{(-2e^{-2} + 8)^2 + (16e^{-2} - 48)^2}$