Tangent Planes and Directional Derivatives

1. Find an equation of the tangent plane for \( z = x \sin(x + y) \) at \((-1, 1)\).

2. Consider the function \( f(x, y) = \frac{2x + 3}{4y + 1} \).
   (a) Find an equation of the tangent plane to the surface \( z = f(x, y) \) at \((0, 0)\).
   (b) Use your equation from part (a) to approximate the value of \( f(0.01, 0.01) \), and find the actual value of \( f(0.01, 0.01) \) rounded to three decimal places to compare.

3. We don’t know the equation that defines a certain surface \( S \), but we are able to determine two tangent vectors at the point \((2, 1, 3)\) on the surface, namely
   \[ \vec{v}_1 = \langle 3, 0, -4 \rangle \quad \text{and} \quad \vec{v}_2 = \langle 1, 6, 2 \rangle. \]
   Using this information, determine an equation for the tangent plane at \((2, 1, 3)\).

4. The function \( A(x, y) = 4000 + 3xy - 4x^2 - 5y^2 \) gives the altitude in feet at any point \((x, y)\) on a hill (we can think of the \((x, y)\) coordinates as essentially giving longitude and latitude). We are currently located on the hill at \((-1, 2)\).
   (a) Find our current altitude.
   (b) Find the initial slope if we start moving in the direction of the vector \( \langle 1, 7 \rangle \)
   (c) Find a nonzero vector that points in a direction in which the initial slope will be 0 (your vector does not have to be a unit vector).

5. Let \( f(x, y) = 8e^{y\sqrt{x}} - x^2y^3 \).
   (a) Find the derivative of \( f \) in the direction \( \langle 5, 3 \rangle \) at the point \((4, -1)\).
   (b) At the point \((4, -1)\), find the direction in which the maximum derivative of \( f \) occurs, and find the maximum derivative of \( f \).
Answers

1. \( z = -x - y \)

2. (a) \( z = 2x - 12y + 3 \)
   (b) Tangent plane approximation 2.9, actual value of 2.904 (rounded to three decimal places)

3. \( 24(x - 2) - 10(y - 1) + 18(z - 3) = 0 \)

4. (a) 3,970 feet
   (b) \( \frac{-147}{\sqrt{50}} \)
   (c) Any constant multiple of \( \langle 23, 14 \rangle \) will work

5. (a) \( \frac{38e^{-2} - 104}{\sqrt{34}} \)
   (b) Direction is \( \langle -2e^{-2} + 8, 16e^{-2} - 48 \rangle \), maximum value is \( \sqrt{(-2e^{-2} + 8)^2 + (16e^{-2} - 48)^2} \)