

Spherical Triple Integrals

1. Compute $\iiint_E (x^2 + y^2 + z^2)^{1/2} dV$, where $E = \{(x, y, z) \mid x \leq 0, y \leq 0, z \geq 0, 4 \leq x^2 + y^2 + z^2 \leq 16\}$.

2. Compute $\iiint_E x dV$, where $E = \{(x, y, z) \mid x \leq 0, z \leq 0, x^2 + y^2 + z^2 \leq 9\}$.

3. Sketch the solid whose volume is given by the iterated integral.

(a) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^3 \rho^2 \sin \phi d\rho d\phi d\theta$

(b) $\int_0^{\pi/6} \int_0^{\pi/2} \int_0^3 \rho^2 \sin \phi d\rho d\theta d\phi$

(c) $\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^2 \rho^2 \sin \phi d\rho d\phi d\theta$

4. Compute the following integral by making a change in coordinates.

$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} x^2 \sqrt{x^2 + y^2 + z^2} dz dx dy.$$

Answers

1. 30π

2. $-\frac{81\pi}{8}$

4. $\frac{64\pi}{9}$