Double Integrals in Polar Coordinates

1. Let D be the region in the first quadrant of the xy-plane given by $1 \leqslant x^2 + y^2 \leqslant 4$. Set up and evaluate a double integral of the function $f(x, y) = xy$ over the region.

2. Evaluate each of the following double integrals by converting to polar coordinates.
   (a) $\displaystyle\int_{-1}^{0}\int_{0}^{\sqrt{1-x^2}} 8x^3 y \, dy \, dx$
   (b) $\displaystyle\int_{0}^{\sqrt{2}}\int_{y}^{\sqrt{4-y^2}} e^{x^2+y^2} \, dx \, dy$

3. Let D be the region in the third quadrant enclosed by $y = x$, $y = 0$, and $x^2 + y^2 = 16$.
   (a) Sketch the region D.
   (b) Write the double integral $\displaystyle\iint_D \frac{3x^2}{y} \, dA$ in polar coordinates (do not evaluate).

4. Suppose we want to find the volume between the planes $z = x - y$ and $z = 0$ inside the cylinder $x^2 + y^2 = 4$.
   (a) Evaluate $\displaystyle\iint_D (x - y) \, dA$ where D is the region $x^2 + y^2 \leqslant 4$. Why is this not the desired volume?
   (b) Try to sketch the volume we are looking for: sketch the plane $z = x - y$, then the cylinder $x^2 + y^2 = 4$. Now use symmetry and a double integral to compute the volume described above.