Lines and Planes

1. Find an equation of the line that passes through the points $(1, 3, -1)$ and $(-2, -1, 5)$.

2. Find an equation of the plane that contains the point $(2, 1, -5)$ and is parallel to the plane with equation $4x - 12z = 7y + 2$.

3. Find all value(s) of t where the line $\vec{r}(t) = \langle 1 - t, t, t \rangle$ intersects the sphere $x^2 + y^2 + z^2 = 4$.

4. Are the planes $2x - y - z = 3$ and $2x + 2y + z = 1$ parallel, orthogonal, or neither? Be sure to justify your answer.

5. Do the four points $P(1, 1, 3)$, $Q(2, -4, -1)$, $R(2, 0, 2)$, and $S(3, -1, 1)$ all lie in a single plane? If so, find an equation of the plane, and if not, explain why not.

6. Find an equation of the plane that contains the line $\vec{r}(t) = \langle 4 - 2t, t, 5 - 4t \rangle$ and the point $(1, 1, 1)$.

7. Determine the equation of two planes whose intersection is the y-axis. Neither plane that you use can be the xy-plane or yz-plane.

Answers

1. One possible answer is $\vec{r}(t) = \langle 1 - 3t, 3 - 4t, -1 + 6t \rangle$.
2. One possible answer is $4(x - 2) - 7(y - 1) - 12(z + 5) = 0$.
3. $t = \dfrac{2 \pm \sqrt{40}}{6}$
4. Neither
5. Yes, and one possible equation is $(x - 1) - 3(y - 1) + 4(z - 3) = 0$.
6. One possible answer is $4y + z = 5$.
7. There are an infinite number of pairs that work. One such pair of planes is $z = x$ and $z = -3x$.