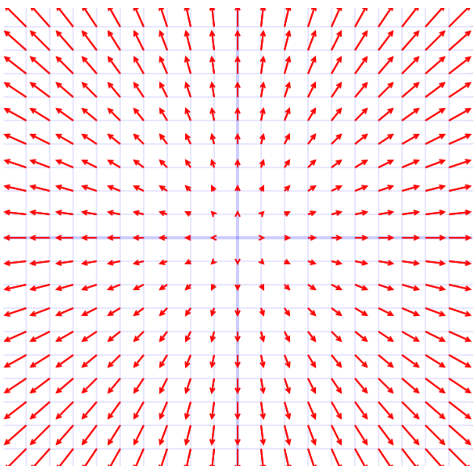
Line Integrals

1. Evaluate the line integral $\int_C \frac{y}{x}\, ds$ if C is the portion of the curve $y = x^2$ from $(1,1)$ to $(2,4)$.

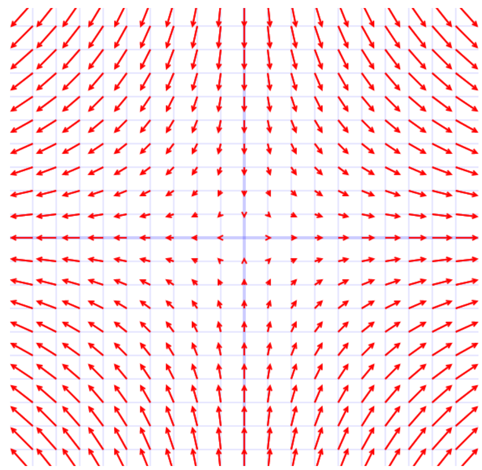
2. Evaluate the line integral $\int_C (x^2 + y^2 + z^2)\, ds$ if C is the curve given by $x = t$, $y = \cos 2t$, $z = \sin 2t$, $0 \leqslant t \leqslant 2\pi$.

3. Evaluate the line integral $\int_C x \sin y\, ds$ if C is the line segment from $(0,3)$ to $(4,6)$.

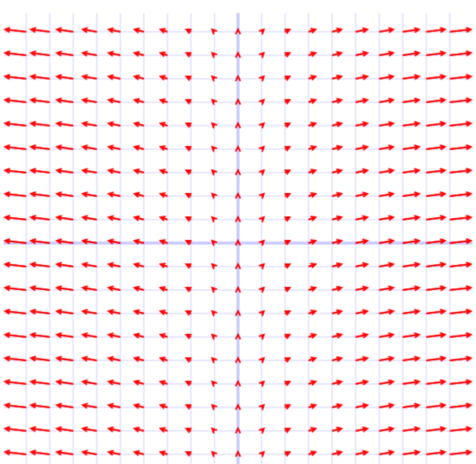
4. Match each plot with one of the vector fields below. Explain how you know you are correct. Images created on https://kevinmehall.net/p/equationexplorer/vectorfield.html.



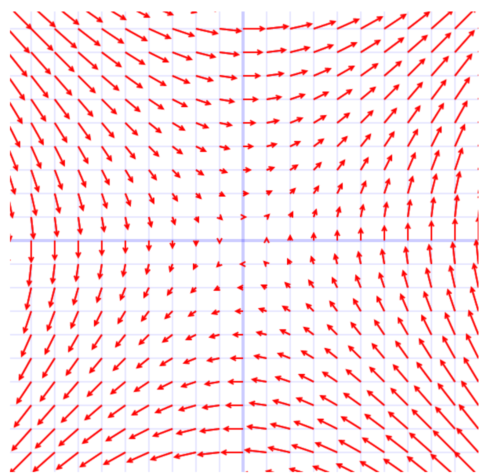
I



II



III



IV

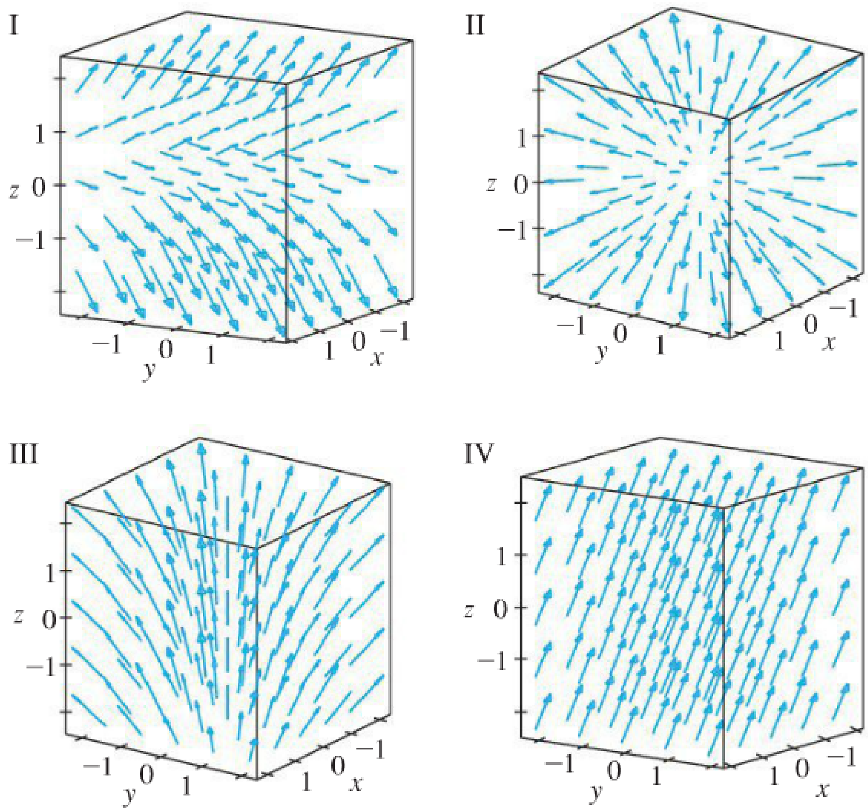
(a) $\vec{F}(x,y) = \langle y, x \rangle$

(b) $\vec{F}(x,y) = \langle x, y \rangle$

(c) $\vec{F}(x,y) = \langle x, -y \rangle$

(d) $\vec{F}(x,y) = \langle x, 1 \rangle$

5. Match each plot with one of the vector fields below. Explain how you know you are correct. Image taken from Stewart (8th Edition).



(a) $\vec{F}(x, y, z) = \langle 1, 2, 3 \rangle$

(b) $\vec{F}(x, y, z) = \langle 1, 2, z \rangle$

(c) $\vec{F}(x, y, z) = \langle x, y, 3 \rangle$

(d) $\vec{F}(x, y, z) = \langle x, y, z \rangle$

6. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F}(x, y) = \langle xy, 3y^2 \rangle$ and C is the curve given by $x = 11t^4$, $y = t^3$, $0 \leqslant t \leqslant 1$.

7. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F}(x, y, z) = \langle x, y, xy \rangle$ and C is the curve given by $x = \cos t$, $y = \sin t$, $z = t$, $0 \leqslant t \leqslant \pi$.

8. Consider the force field $\vec{F}(x, y) = \langle x^2, xy \rangle$.

   (a) If a particle moves once around the circle $x^2 + y^2 = 4$ in the counter-clockwise direction, find the work done by the given force field on the particle.

   (b) Does anything change if the particle moves around the circle in the clockwise direction instead? Explain.

9. Show that a constant force field does zero work on a particle that moves once around the circle $x^2 + y^2 = 1$. Is the work still zero if the radius is not 1? Explain.

Answers

1. $\frac{1}{12}\left[17^{3/2} - 5^{3/2}\right]$
2. $\sqrt{5}\left(\frac{8\pi^3}{3} + 2\pi\right)$
3. $-\frac{20}{3}\cos(6) + \frac{20}{9}\sin(6) - \frac{20}{9}\sin(3)$
4. (a) IV
   (b) I
   (c) II
   (d) III
5. (a) IV
   (b) I
   (c) III
   (d) II
6. 45
7. 0
8. (a) 0
   (b) No