

Double Integrals and Volume

1. Evaluate each of the following double integrals.

(a) $\int_1^2 \int_1^4 \left(\frac{2x}{y} - \frac{1}{\sqrt{y}} \right) dy dx$

(b) $\int_3^4 \int_1^5 \frac{x \ln y}{y} dx dy$

(c) $\int_0^1 \int_0^{\pi/6} xy \cos(3x) dx dy$

2. (a) Find the volume contained beneath $z = 3x + 2y$ and above the xy -plane over the rectangle $0 \leq x \leq 1$, $0 \leq y \leq 2$.

(b) Find the volume contained between the surfaces $z = 3x + 2y$ and $z = -x^2 - y^2$ over the same rectangle.

3. Let D be the region in the xy -plane bounded by $x = 0$, $y = 1 - x$, and $y = 2 - 2x$.

(a) Sketch the region D , and say whether it is type x , type y , both, or neither.

(b) Set up, **but do not evaluate**, one double integral to integrate $f(x, y) = xy$ over the region D .

4. Let D be the region in the first quadrant of the xy -plane given by $1 \leq x^2 + y^2 \leq 4$.

(a) Sketch the region D , and say whether it is type x , type y , both, or neither.

(b) Set up, **but do not evaluate**, a double integral or sum of double integrals to integrate $f(x, y) = xy$ over the region D .

5. Consider the double integral $\int_0^5 \int_y^5 e^{(x^2)} dx dy$.

(a) Sketch the region being integrated over.

(b) Rewrite the integral in the opposite order of integration, then evaluate. Notice that we cannot evaluate the integral by hand when it is written in the original order.

Answers

1. (a) $3\ln(4) - 2$
(b) $6(\ln(4))^2 - 6(\ln(3))^2$
(c) $\frac{\pi}{36} + \frac{1}{18}$
2. (a) 7
(b) $\frac{31}{3}$
3. (a) D is a triangle with vertices $(0, 1)$, $(0, 2)$, and $(1, 0)$, which is type y only.
4. (a) D is the quarter of the annulus (ring) in the first quadrant with inner radius 1 and outer radius 2, which is neither type x nor type y .
5. (a) The region is a triangle with vertices $(0, 0)$, $(5, 0)$, and $(5, 5)$.
(b) $\frac{1}{2}(e^{25} - 1)$