Classifying Critical Points

1. Find and classify all critical points of the function $f(x, y) = x^3 - 6xy - y^2$

2. Find and classify all critical points of the function $f(x, y) = 5xye^{-y^2}$.

3. Show that the function $f(x, y) = x^2 + 4y^2 - 4xy + 2$ has an infinite number of critical points and that $D = 0$ at each one.

4. For a function of one variable, it is impossible for a continuous function to have, for example, two local maxima without a local minimum (or vice versa). However, for functions of two or more variables, such functions exist. Given the function

$$f(x, y) = -(x^2 - 1)^2 - (x^2y - x - 1)^2,$$

show that $(-1, 0, 0)$ and $(1, 2, 0)$ are critical points and that both are local maxima. (In fact, these two points are the only critical points for this function).

Answers

1. $(0, 0)$ is a saddle point, $(-6, 18)$ is a local maximum
2. $(0, 0)$ is a saddle point
3. Any point on the line $x = 2y$ is a critical point with $D = 0$