

Cartesian Triple Integrals

1. Compute the iterated integral.

(a) $\int_0^2 \int_0^1 \int_0^3 (xy + z^2) dz dy dx$

(b) $\int_0^{\pi/2} \int_0^y \int_0^x \cos(x + y + z) dz dx dy$

2. Compute $\iiint_E xy dV$, where $E = \{(x, y, z) \mid 0 \leq x \leq 3, 0 \leq y \leq x, 0 \leq z \leq x + y\}$.

3. Compute $\iiint_E x^2 dV$, where E is the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

4. Sketch the solid whose volume is given by the iterated integral.

(a) $\int_0^1 \int_0^{1-x} \int_0^{2-2z} dy dz dx$

(b) $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx dz dy$

5. Write the other 5 integrals that are equivalent to

$$\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} f(x, y, z) dz dy dx.$$

Answers

1. (a) 21

(b) $-\frac{1}{3}$

2. $\frac{243}{6}$

3. $\frac{1}{6} - \frac{1}{4} + \frac{1}{10}$