Absolute Extrema Practice Exercises Solutions

1. $f_x = -2x = 0 \Rightarrow x = 0$

   $f_y = 2y = 0 \Rightarrow y = 0$

   $(0,0)$ is a critical point inside the given region

   Constraint is $g(x,y) = x^2 + 4y^2 = 4$. Lagrange multipliers: $\vec{\nabla} f = \lambda \vec{\nabla} g$.

   $$\begin{aligned} -2x &= \lambda 2x \\ 2y &= \lambda 8y \\ x^2 + 4y^2 &= 4 \end{aligned}$$

   If $x \neq 0$ an $y \neq 0$, then $\lambda = -1$ and $\lambda = \frac{1}{4}$, impossible, so must have $x = 0$ or $y = 0$.

   If $x = 0$ and $x^2 + 4y^2 = 4$, then $y = \pm 1$, so $(0,1)$ and $(0,-1)$ are possible points.
   If $y = 0$ and $x^2 + 4y^2 = 4$, then $x = \pm 2$, so $(2,0)$ and $(-2,0)$ are possible points.

   $f(0,0) = 0$

   $f(0,1) = 1$ (absolute maximum value)

   $f(0,-1) = 1$ (absolute maximum value)

   $f(2,0) = -4$ (absolute minimum value)

   $f(-2,0) = -4$ (absolute minimum value)

2. Area $= A(x,y) = xy$

   The only critical point is $(0,0)$, but it doesn't make sense in the context of the problem, so it can be ignored.

   Constraint is perimeter $P(x,y) = 2x + 2y = 14$. Lagrange multipliers: $\vec{\nabla} A = \lambda \vec{\nabla} P$.

   $$\begin{aligned} y &= 2\lambda \\ x &= 2\lambda \\ 2x + 2y &= 14 \end{aligned}$$

   We have $x = 2\lambda = y$ so $x + x = 7$ and $x = \frac{7}{2}$, so $y = \frac{7}{2}$.

   $A(\frac{7}{2}, \frac{7}{2}) = \frac{49}{4}$

   Pick any other point (x,y) satisfying the constraint, for example $(1,6)$: $A(1,6) = 6 < \frac{49}{4}$. Thus, $\frac{49}{4}$ is the maximum value. There is no minimum value if $x \neq 0$ and $y \neq 0$.