Understanding Surfaces

1. (a) Consider the sphere $x^2 + y^2 + z^2 = 8$, and the half cone $z = \sqrt{x^2 + y^2}$ (the full cone is $z^2 = x^2 + y^2$, and the other half is $z = -\sqrt{x^2 + y^2}$). Find the intersection of these two surfaces—what kind of curve is it? can we say how large it is?

   (b) Find the equation of a sphere whose intersection with the half cone $z = \sqrt{x^2 + y^2}$ is a single point.

   (c) Find the equation of a sphere that does not intersect the half cone $z = \sqrt{x^2 + y^2}$.

2. For each of the following, describe all possible intersections of the given surfaces. If it is possible for the surfaces to not intersect, be sure to mention this as well.

   (a) Two planes

   (b) A plane and a circular cylinder

   (c) A plane and a sphere

   (d) Two spheres

   (e) An elliptic paraboloid and a plane

   (f) A (true) cone and a plane

3. What is the difference between $x^2 + y^2 = 9$ and $x^2 + y^2 \leqslant 9$? What do they look like? Do they both have surface area? volume?

4. Sketch the region in $\mathbb{R}^3$ given by $1 \leqslant x^2 + y^2 \leqslant 4$, $z \leqslant 0$.

5. Assume that gravity acts in the direction of the negative z-axis. If we poured water from high above the xy-plane, which of the surfaces $z = x^2$, $z = y^2$, $x = y^2$, and $y = z^2$ would hold the water? Why or why not?

Answers

1. (a) A circle of radius 2
   (b) Answers will vary. One example is $x^2 + y^2 + (z+2)^2 = 4$.
   (c) Answers will vary. One example is $x^2 + y^2 + (z-10)^2 = 1$.
2. (a) No intersection, line, plane
   (b) No intersection, one line, pair of parallel lines, circle, ellipse
   (c) No intersection, point, circle
   (d) No intersection, point, circle, sphere
   (e) No intersection, point, parabola, ellipse (maybe circles too, depending)
   (f) Point, line, parabola, ellipse, hyperbola (must intersect)
3. $x^2 + y^2 = 9$ is a hollow cylinder of radius 3 that has surface area but $x^2 + y^2 \leqslant 9$ is a solid cylinder of radius 3 that has both surface area and volume.
4. Bottom/lower half of a solid cylinder (tube) along the z-axis with inner radius 1 and outer radius 2.
5. $z = x^2$ and $z = y^2$