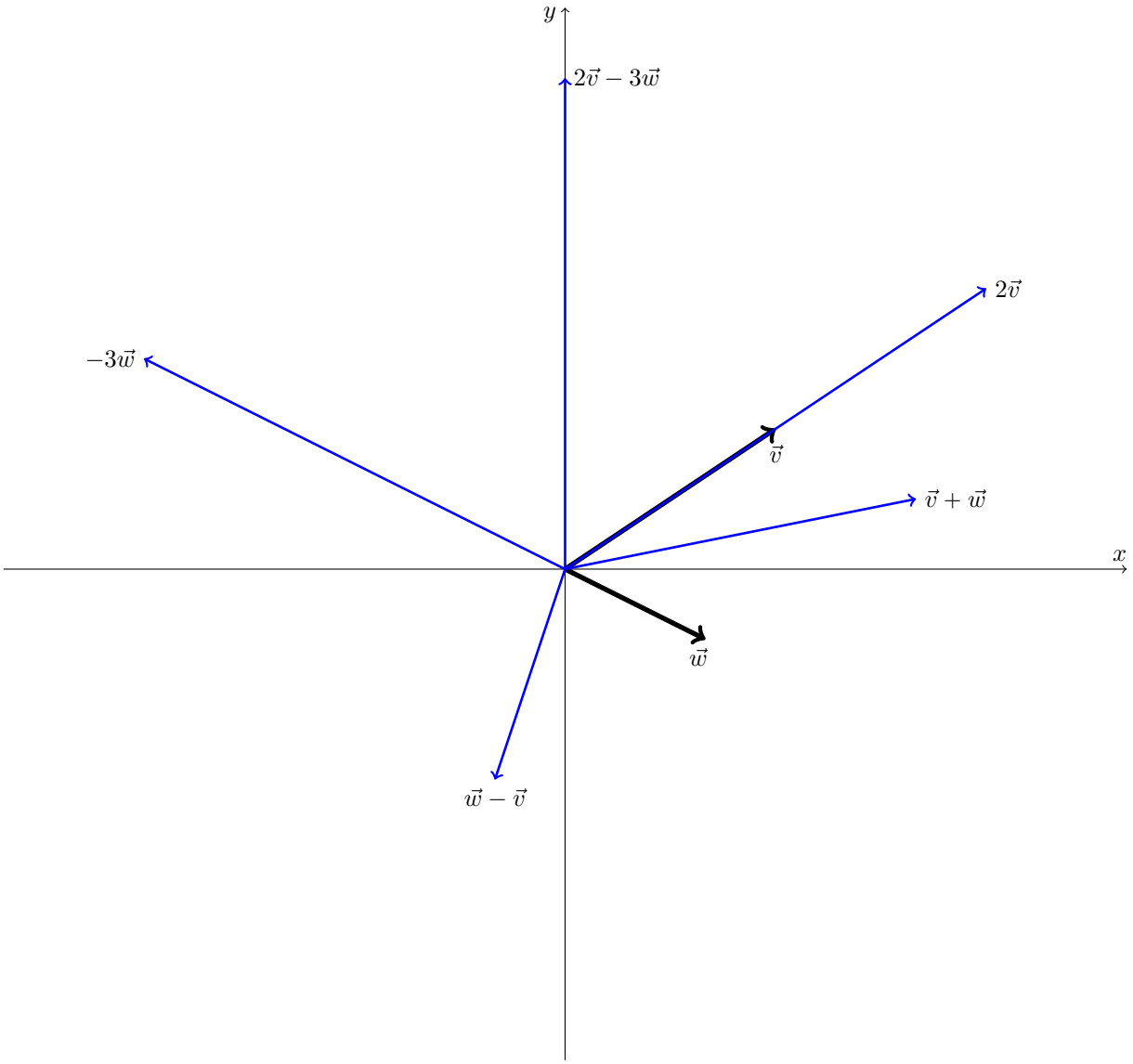
Practice with Vectors Solutions

1. On the given set of axes, sketch $\vec{v}+\vec{w}$, $\vec{w}-\vec{v}$, $2\vec{v}$, $-3\vec{w}$, and $2\vec{v}-3\vec{w}$.



2. $\vec{v}-\vec{w}=\langle 3,3,-3\rangle$

$$\vec{u}=\frac{\vec{v}-\vec{w}}{|\vec{v}-\vec{w}|}=\frac{\langle 3,3,-3\rangle}{\sqrt{9+9+9}}=\left\langle \frac{3}{\sqrt{27}},\frac{3}{\sqrt{27}},-\frac{3}{\sqrt{27}}\right\rangle.$$

3. (a) $\cos\theta=\dfrac{\langle 2,7\rangle\cdot\langle 3,-1\rangle}{|\langle 2,7\rangle||\langle 3,-1\rangle|}=\dfrac{-1}{\sqrt{53}\sqrt{10}}$ so $\theta=1.61$ rad (2 dec plcs)

   (b) $\cos\theta=\dfrac{-15-12+2}{\sqrt{14}\sqrt{65}}$ so $\theta=2.55$ rad (2 dec plcs).

4. Find a vector that is orthogonal to each given pair of vectors.
   (a) $\vec{a} \times \vec{b} = (4+18)\hat{\mathbf{i}} + (18-2)\hat{\mathbf{j}} + (-6-12)\hat{\mathbf{k}} = 22\hat{\mathbf{i}} + 16\hat{\mathbf{j}} - 18\hat{\mathbf{k}}$.
   (b) $\langle 10, 5, -3\rangle \times \langle 4, 7, 2\rangle = \langle 10+21, -12-20, 70-20\rangle = \langle 31, -32, 50$

5. Direction from Bond to Green: $\langle -4, -2\rangle - \langle 1, 6\rangle = \langle -5, -8\rangle$
   $\cos\theta = \dfrac{\langle -5, -8\rangle \cdot \langle -1, 2\rangle}{|\langle -5, -8\rangle||\langle -1, 2\rangle|} = \dfrac{-11}{\sqrt{89}\sqrt{5}}$ so $\theta = 2.12$ rad (2 dec plcs).

6. Displacement $\vec{D} = \langle 8-4, 1+3, 7+3\rangle = \langle 4, 4, 10\rangle$
   $W = \vec{F} \cdot \vec{D} = -24 + 4 + 110 = 90$ Joules.

7. (a) For example, $\vec{b} = \langle 1, 2, -3, \rangle$
   (b) Let $\vec{c} = \vec{a} \times \vec{b} = \langle -15-4, 2-12, -8-5\rangle = \langle -19, -10, -13\rangle$

8. $0 = \langle 1, -3, 2\rangle \cdot \langle 2a, a, 3\rangle = -a + 6$ so $a = 6$.