Calclulus I and II Review

You should be able to answer all of the following questions. If you cannot remember how to solve a given problem, it would be wise to read the relevant section(s) in Stewart and try again. Understanding these concepts and computations is crucial to your success in this course.

1. Find the derivative of each of the following functions.
   (a) $f(x) = x^5 - 4x^{3/4} + 2x^{-1} - 3$
   (b) $g(x) = \ln(\cos(2x))$
   (c) $y = \dfrac{3x}{x^2+1}$
   (d) $h(x) = 3x^2y + 2xe^{xt}$ (x is the only variable here, treat the rest as constants)
   (e) $F(x) = \displaystyle\int_0^x 18e^{-2t}\sin(3t^2-5)\,dt$
2. Find $\dfrac{dy}{dx}$ if $xy^2 + e^y - 3x = x^3$.
3. Find the equation of the tangent line at $x = 0$ for the function $f(x) = \big(\tan(x)+1\big)^4$.
4. Find and classify all critical points for the following functions.
   (a) $f(x) = x^3 - x$
   (b) $y = xe^{-x}$
5. A function $f(x)$ has critical values $x = -2, 0, 1$. If $f''(x) = \dfrac{x}{x-3}$, classify each critical value as a local maximum, local minimum, or neither.
6. Determine the absolute maximum and minimum values of the function $f(x) = x^2 - 4x + 1$ on the interval $[1, 5]$.
7. Evaluate the given indefinite integrals.
   (a) $\displaystyle\int (x^2 + \frac{1}{\sqrt{x}} - 4x^{-1} + 1)\,dx$
   (b) $\displaystyle\int \frac{2}{x+1}\,dx$
   (c) $\displaystyle\int (x+2)e^{3x}\,dx$
   (d) $\displaystyle\int \frac{4}{x^2+1}\,dx$
   (e) $\displaystyle\int \cos^4 x \sin x\,dx$
   (f) $\displaystyle\int \frac{4x^2}{x^3+9}\,dx$
   (g) $\displaystyle\int \frac{(\ln x)^5}{4x}\,dx$

8. Find the area of the given region.
   (a) The region in the first quadrant bounded by $y = x^2$, $y = 0$ and $x = 3$.
   (b) The region contained between $f(x) = 2x + 8$ and $g(x) = x^2 + 5$.
   (c) The region contained between $y = \sqrt{4 - x^2}$ and $y = 0$.
   (d) The region bounded by $x = 1 - y^2$ and $x = -3$.

9. Write a definite integral that could be used to find the area of a right triangle with sides of length 2 and 5.

10. Rewrite the given equation in polar coordinates.
   (a) $x = 4$
   (b) $(x^2 + y^2)^{3/2} = y$
   (c) $x^2 + 6x + y^2 = -5$

11. Sketch a graph of the curve given by the following parametric equations. Indicate the direction of motion as $t \to \infty$.
   (a) $x = t + 1,\ y = 3t$
   (b) $x = t,\ y = t^3$
   (c) $x = 1 + t^2,\ y = 2t$
   (d) $x = 3\cos t,\ y = 3\sin t$
   (e) $x = 2\cos t,\ y = 5\sin t$