Exam 2 Practice Problems

These questions are not meant to be an exhaustive review and will not provide sufficient practice for the exam, but they should help you begin studying and to diagnose material that you need to practice most.

1. True or False? The following triple integral represents the volume of the region bounded by the surfaces $z = (x^2 + y^2)^{3/2}$ and $z = 0$ inside the cylinder $x^2 + y^2 = 4$.

$$\int_0^{2\pi} \int_0^2 \int_0^8 r^4 \, dz \, dr \, d\theta$$

2. True or False? Let D be a region in the xy-plane that is enclosed by a simple, closed curve C traced once counterclockwise. Then the value of the line integral of $\vec{F} = \langle 2y, 3x \rangle$ over C will yield the area of the region D.

3. True or False? The intersection of the surfaces $\rho = 3\sec(\phi)$ and $\rho = 5\csc(\phi)$ is a circle.

4. Compute the arc length of the curve given by $\vec{r}(t) = \langle \frac{4}{3}t^{3/2}, \frac{1}{2}t^2 - 2t, \frac{4}{3}t^{3/2} \rangle$ from $t = 1$ to $t = 2$.

5. Set up and evaluate a triple integral in Cartesian coordinates to find the volume enclosed by $y = 0$, $z = 0$, $y = \sqrt{x}$, and $z = 1 - x$.

6. Set up but **do not evaluate** a triple integral to find the volume enclosed between the xy-plane and $z = 2(x^2 + y^2) + 3$ over the region D in the *second quadrant* enclosed by $x^2 + y^2 = 25$ using
   (a) Cartesian coordinates.
   (b) Cylindrical coordinates.

7. Compute the line integral of $f(x, y) = 4xy$ over the line segment from $(1, -2)$ to $(3, 0)$.

8. Compute the line integral of $\vec{F} = \langle y - x, x^3 \rangle$ over the portion of the curve $y = x^2$ from $(-1, 1)$ to $(1, 1)$.

9. Let $\vec{F} = \langle 3x^2 - 2xy + 5, y^3 - x^2 \rangle$ be a vector field.
   (a) Show that $\vec{F}$ is conservative.
   (b) Find a function $f(x, y)$ so that $\vec{F} = \vec{\nabla} f$.
   (c) If C is the circle $(x - 2)^2 + (y + 4)^2 = 9$ traced once clockwise, evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$.

10. Let C be the path consisting of the line segment from $(0, 0)$ to $(1, 1)$, followed by the portion of the circle of radius $\sqrt{2}$ traced counterclockwise from $(1, 1)$ to $(1, -1)$, followed by the line segment from $(1, -1)$ to $(0, 0)$. Sketch the curve C (be sure to indicate the orientation), then use Green's Theorem to evaluate the line integral of $\vec{F} = \langle x^2 - xy, e^{cos(y)} \rangle$ over C.

11. Write the following integral using **spherical coordinates** if the solid E is bounded below by $z = \sqrt{x^2 + y^2}$ and above by $z = 1$. **Do not evaluate**.

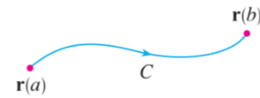
$$\iiint_E y^2 z \, dV$$

Fundamental Theorem of Calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

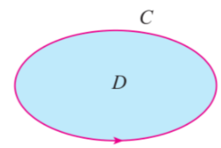
Fundamental Theorem for Line Integrals

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$



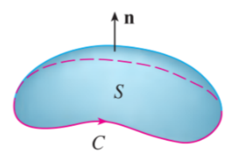
Green's Theorem

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_C P\,dx + Q\,dy$$



Stokes' Theorem

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$$



Divergence Theorem

$$\iiint_E \operatorname{div} \mathbf{F}\,dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$$