Exam 2 Practice Problems Solutions

1. $z=(x^2+y^2)^{\frac{3}{2}}=(r^2)^{\frac{3}{2}}=r^3$, so

$$V=\iiint_E 1\,dV=\int_0^{2\pi}\int_0^2\int_0^{r^3} r\,dz\,dr\,d\theta \Rightarrow \underline{\text{False.}}$$

2. C is a simple, closed curve, so we can apply Green's Theorem.

$$\vec{F}=\langle 2y,3x\rangle \Rightarrow \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=3-2=1, \text{ so}$$

$$A(D)=\iint_D 1\,dA=\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA=\oint_C \vec{F}\cdot d\vec{r} \Rightarrow \underline{\text{True}}$$

3. $\rho=3\sec\phi$ is a ~~cylinder~~ plane (flat plane)

$(\rho=3\sec\phi \Rightarrow \rho\cos\phi=3 \Rightarrow z=3)$

and $\rho=5\csc\phi$ is a cylinder

$(\rho=5\csc\phi \Rightarrow \rho\sin\phi=5 \Rightarrow x^2+y^2=25)$

so the intersection of the surfaces is a circle

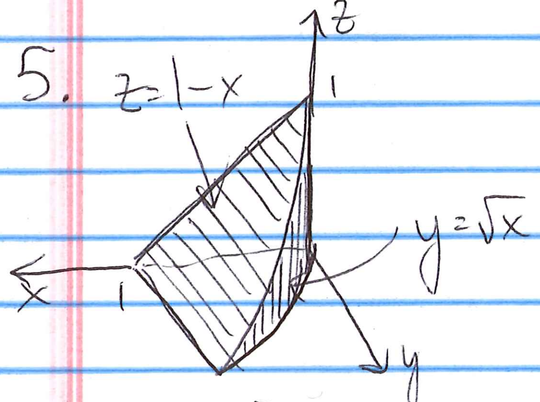
$\Rightarrow$ <u>True.</u>

4. $\vec{r}'(t) = \langle 2t^{\frac{1}{2}}, t-2, 2t^{\frac{1}{2}} \rangle$

$$\Rightarrow |\vec{r}'(t)| = \sqrt{4t+(t-2)^2+4t} = \sqrt{t^2+4t+4} = t+2$$

$$\therefore L = \int_a^b |\vec{r}'(t)|\, dt = \int_1^2 (t+2)\, dt = \underline{\frac{7}{2}}.$$

5.



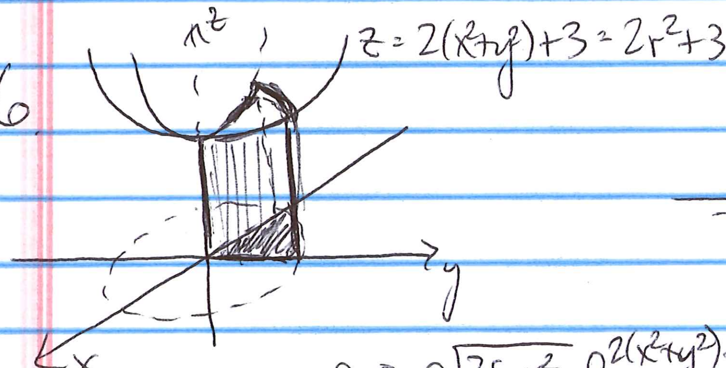
$$V = \iiint_E 1\, dV$$

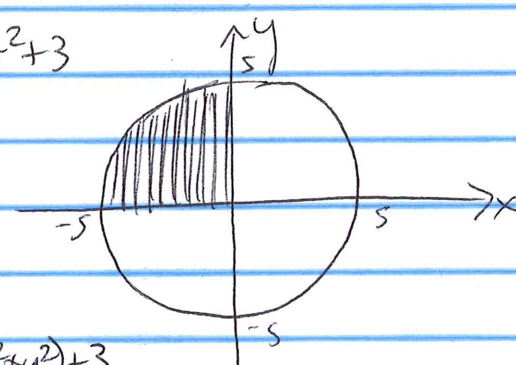
$$= \int_0^1 \int_0^{\sqrt{x}} \int_0^{1-x} 1\, dz\, dy\, dx$$

$$= \int_0^1 \int_0^{\sqrt{x}} (1-x)\, dy\, dx = \int_0^1 (1-x)\sqrt{x}\, dx$$

$$= \int_0^1 \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx = \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right)\Big|_0^1 = \underline{\frac{2}{3} - \frac{2}{5}} = \underline{\frac{4}{15}}$$

6.





Cartesian: $\int_{-5}^{0} \int_0^{\sqrt{25-x^2}} \int_0^{2(x^2+y^2)+3} 1\, dz\, dy\, dx$

Cylindrical: $\int_{\frac{\pi}{2}}^{\pi} \int_0^5 \int_0^{2r^2+3} r\, dz\, dr\, d\theta$

7. $C: \vec{r}(t) = \langle 1, -2 \rangle + t\langle 2, 2 \rangle = \langle 1+2t, -2+2t \rangle, \quad 0 \le t \le 1$

$\Rightarrow \vec{r}\,'(t) = \langle 2, 2 \rangle \Rightarrow |\vec{r}\,'(t)| = \sqrt{4+4} = \sqrt{8}$

$\therefore \int_C f\, ds = \int_0^1 4(1+2t)(-2+2t) \cdot \sqrt{8}\, dt$

$= 8\sqrt{8} \int_0^1 (2t^2 - t - 1)\, dt = 8\sqrt{8}\left(\frac{2}{3} - \frac{1}{2} - 1\right) = \underline{\frac{-20\sqrt{8}}{3}}$

8. Note: $\vec{F}$ is <u>not</u> conservative! $\frac{\partial Q}{\partial x} = 3x^2 \neq \frac{\partial P}{\partial y} = 1$

$C: \vec{r}(t) = \langle t, t^2 \rangle, \quad -1 \le t \le 1$

$\Rightarrow \vec{r}\,'(t) = \langle 1, 2t \rangle$

$\therefore \int_C \vec{F} \cdot d\vec{r} = \int_{-1}^1 \langle t^2 - t, t^3 \rangle \cdot \langle 1, 2t \rangle\, dt$

$= \int_{-1}^1 (t^2 - t + 2t^4)\, dt = 2\int_0^1 (t^2 + 2t^4)\, dt = 2\left(\frac{1}{3} + \frac{2}{5}\right) = \underline{\frac{22}{15}}$

int is from $-a$ to a

even functions

odd function

e.g. $\left(\int_{-a}^{a} x\, dx = 0\right)$

9. a) $\frac{\partial Q}{\partial x} = -2x = \frac{\partial P}{\partial y} \Rightarrow \vec{F}$ is conservative

b) $f(x,y) = \int (3x^2 - 2xy + 5)\,dx = x^3 - x^2 y + 5x + g(y)$

so $f_y = -x^2 + g'(y)$, but we also know $f_y = y^3 - x^2$,

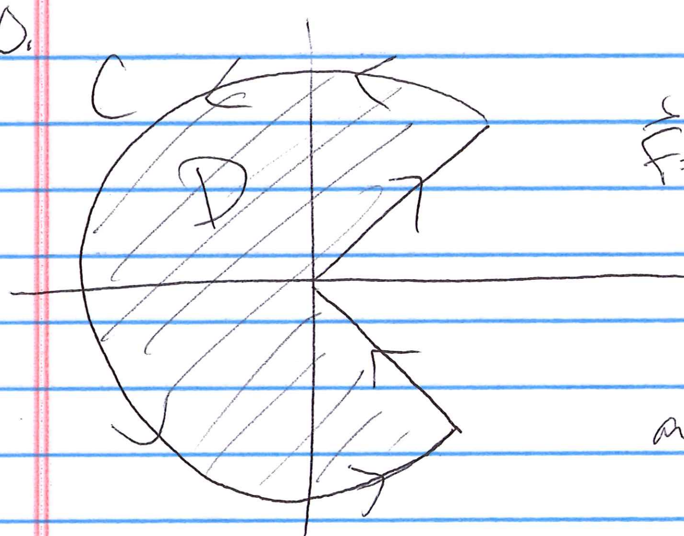
therefore $g'(y) = y^3$ and $g(y) = \frac{1}{4}y^4$

$\therefore \underline{f(x,y) = x^3 - x^2 y + 5x + \frac{1}{4}y^4}$ (one possible choice)

c) Since $\vec{F} = \vec{\nabla} f$ is conservative and the curve C is closed, we have

$$\underline{\int_C \vec{F} \cdot d\vec{r} = 0}$$

10.



$\vec{F} = \langle P, Q \rangle = \langle x^2 - xy, e^{\cos y} \rangle$

$\Rightarrow \frac{\partial Q}{\partial x} = 0, \ \frac{\partial P}{\partial y} = -x$

and $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = x.$

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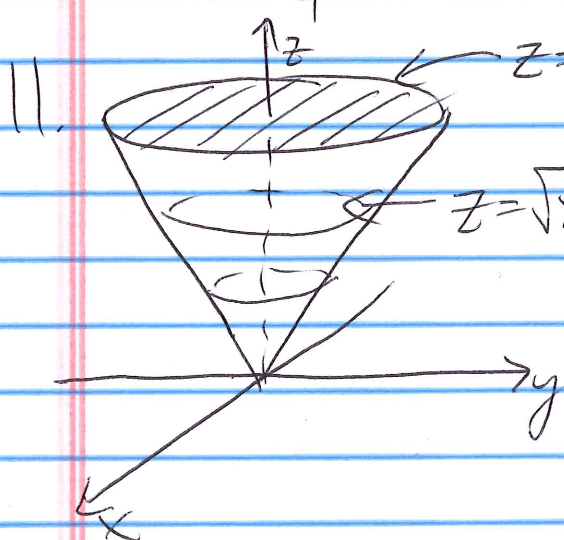
$$\therefore \oint_C \vec{F}\cdot d\vec{r} \overset{\text{Green's Theorem}}{=} \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \iint_D x \, dA \quad \left(\begin{array}{c}D \text{ is nicely described} \\ \text{in polar coordinates}\end{array}\right)$$

$$= \int_{\frac{\pi}{4}}^{\frac{7\pi}{4}} \int_0^{\sqrt{2}} (r\cos\theta)\cdot r \, dr \, d\theta$$

$$= \left(\int_{\frac{\pi}{4}}^{\frac{7\pi}{4}} \cos\theta \, d\theta\right)\left(\int_0^{\sqrt{2}} r^2 \, dr\right) = \underline{-\frac{4}{3}}$$

11.



$z=1 \Rightarrow \rho\cos\phi = 1 \Rightarrow \rho = \sec\phi$

$z=\sqrt{x^2+y^2} \Rightarrow \phi = \frac{\pi}{4}$

$$\therefore \iiint_E y^2 z \, dV = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sec\phi} (\rho\sin\phi\sin\theta)^2(\rho\cos\phi)\,\rho^2\sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sec\phi} \rho^5 \sin^3\phi\cos\phi\sin^2\theta \, d\rho\, d\phi\, d\theta$$