Exam 1 Practice Problems Solutions

1. $x = 1+t,\ y = 1-t,\ z = 3t$ and $x+2y-z = 11$

$\Rightarrow (1+t)+2(1-t)-3t = 3-4t = 11 \Rightarrow t = -2$

$\vec{r}(-2) = \langle 1-2, 1-(-2), 3(-2)\rangle = \langle -1, 3, -6\rangle \Rightarrow$ True

2. $f(x,y) = e^x\cos(xy)$

$f_x = e^x\cos(xy) - y e^x \sin(xy) \Rightarrow f_x(0,\pi) = 1$

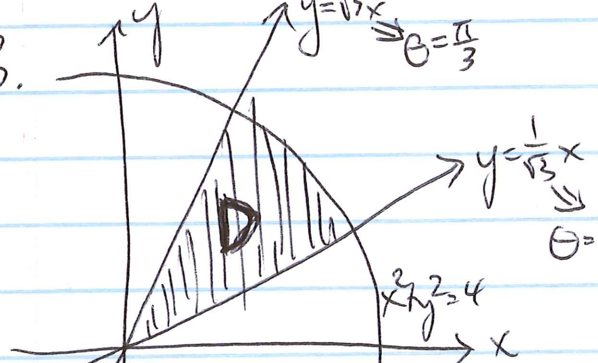
$f_y = -xe^x\sin(xy) \Rightarrow f_y(0,\pi) = 0$

Therefore, the tangent plane at $(0,\pi)$ is given by

$$z = 1 + 1(x-0) + 0(y-\pi) = 1+x$$

So, $f(.1, \pi-.1) \approx 1+.1 = 1.1 \Rightarrow$ False

3.



$$A(D) = \iint_D 1\, dA$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \int_0^2 r\, dr\, d\theta$$

$\Rightarrow$ False

4. $2\vec{a}-3\vec{b} = \langle 10,-2,4\rangle - \langle 9,3,3\rangle = \underline{\langle 1,-5,1\rangle}$

$\vec{a}\cdot\vec{b} = 5\cdot 3 - 1\cdot 1 + 2\cdot 1 = 15-1+2 = \underline{16}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = \langle -1-2, -(5-6), 5+3\rangle = \underline{\langle -3,1,8\rangle}$$

$\vec{a}$ and $\vec{b}$ are not parallel ($\vec{a}\neq k\vec{b}$ for some constant k, also $\vec{a}\times\vec{b}\neq\vec{0}$) and are not orthogonal ($\vec{a}\cdot\vec{b}\neq 0$).

5. $\vec{n}_1 = \langle 1,1,0\rangle$ and $\vec{n}_2 = \langle 1,1,\sqrt{2}\rangle$

$\vec{n}_1\cdot\vec{n}_2 = 1+1+0 = 2$, $|\vec{n}_1| = \sqrt{2}$, and $|\vec{n}_2| = 2$,

so $\cos\theta = \dfrac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \dfrac{2}{\sqrt{2}\cdot 2} = \underline{\dfrac{1}{\sqrt{2}}}$ (and therefore $\theta = \frac{\pi}{4}$)

6. $\vec{\nabla} f = \langle f_x, f_y\rangle = \langle -x^2-y+2, y-x\rangle = \vec{0}$

$\Rightarrow y = x$ and $y = 2-x^2$

$\Rightarrow x = 2-x^2 \Rightarrow x^2+x-2 = (x+2)(x-1) = 0 \Rightarrow x = -2, 1$

$\therefore$ the critical points are $(-2,-2)$ and $(1,1)$.

$f_{xx} = -2x$, $f_{yy} = 1$, $f_{xy} = -1 \Rightarrow D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = -2x-1$.

$D(-2,-2) = 3 > 0$ and $f_{xx}(-2,-2) = 4 > 0 \Rightarrow \underline{(-2,-2) \text{ is a local min}}$

$D(1,1) = -3 < 0 \Rightarrow \underline{(1,1) \text{ is a saddle point}}$

7. To be parallel to the plane $3x-5y+z=10$, the direction vector of the line $\vec{v}$ should be orthogonal to the normal vector of the plane $\vec{n} = \langle 3,-5,1\rangle$.

An infinite number of vectors $\vec{v}$ work, but let's say $\vec{v} = \langle 1,1,2\rangle$.

Check: $\vec{v}\cdot\vec{n} = \langle 1,1,2\rangle \cdot \langle 3,-5,1\rangle = 3-5+2 = 0$ ✓

To finish the line, we need any point, say $(e,\pi,1)$.

$\therefore \vec{r}(t) = \langle e,\pi,1\rangle + t\langle 1,1,2\rangle = \langle e+t, \pi+t, 1+2t\rangle$

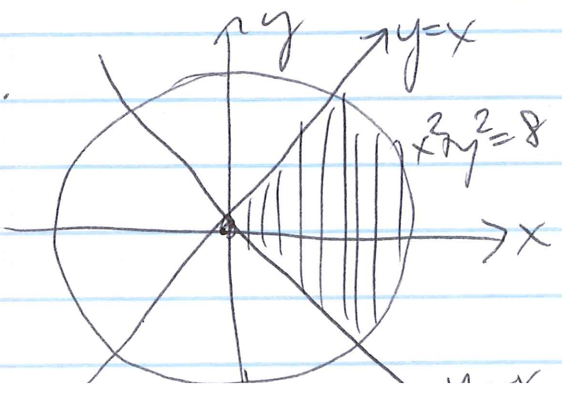
8. $f(1,1) = 7$ (given)

$f_x = 2x(y^3+1)^2+3 \Rightarrow f_x(1,1) = 2\cdot 4+3 = 11$

$f_y = 6x^2y^2(y^3+1) \Rightarrow f_y(1,1) = 6\cdot 2 = 12$

$\therefore z = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1) = 7+11(x-1)+12(y-1)$

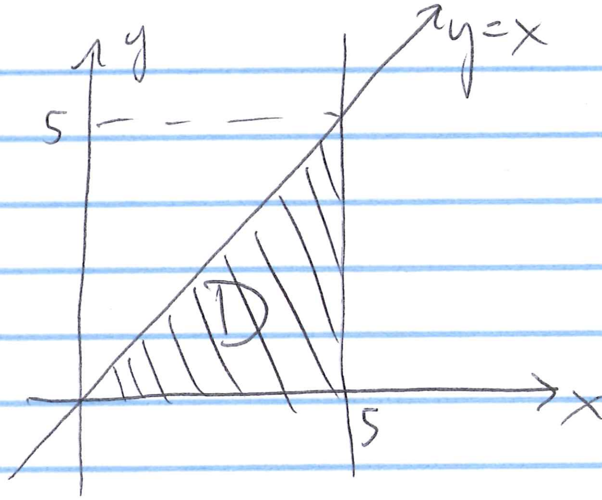
9.



$$A(D) = \iint_D 1\, dA$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{\sqrt{8}} r\, dr\, d\theta$$

10. a)

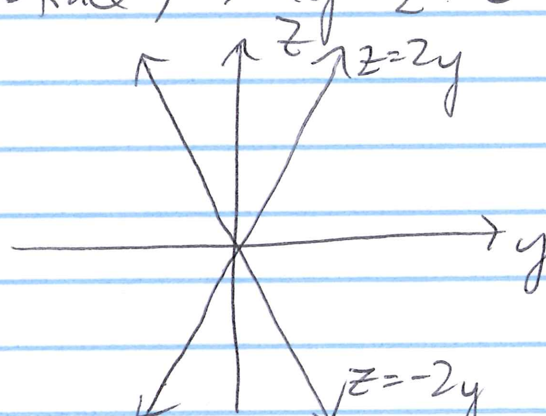


b) Cannot integrate in the given order.

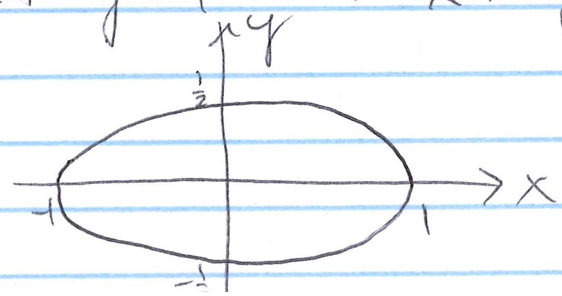
$$\int_0^5 \int_y^5 e^{x^2}\,dx\,dy = \int_0^5 \int_0^x e^{x^2}\,dy\,dx$$

$$= \int_0^5 x e^{x^2}\,dx = \frac{1}{2} e^{x^2} \Big|_0^5 = \underline{\frac{1}{2}\left(e^{25}-1\right)}$$

11. $x=0$ (yz-trace) $\Rightarrow 4y^2 - z^2 = 0 \Rightarrow z^2 = 4y^2 \Rightarrow z = \pm 2y$



$z=1 \Rightarrow x^2+4y^2-1=0 \Rightarrow x^2+4y^2=1$ (ellipse)



12. a) $f(x,y) = x^2 e^{xy} \Rightarrow \vec{\nabla} f = \langle 2xe^{xy} + x^2 y e^{xy}, x^3 e^{xy} \rangle$

$\therefore \vec{\nabla} f(2,0) = \langle 4+0, 8 \rangle = \langle 4, 8 \rangle$

$\vec{u}$ is already given as a unit vector, so

$D_{\vec{u}} f(2,0) = \vec{\nabla} f(2,0) \cdot \vec{u} = \langle 4,8 \rangle \cdot \langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \rangle$

$= \frac{4}{\sqrt{10}} + \frac{24}{\sqrt{10}} = \underline{\frac{28}{\sqrt{10}}}$

b) $\vec{\nabla} f(1,1) = \langle 2e + e, e \rangle = \langle 3e, e \rangle$

$\Rightarrow$ Direction of maximum derivative is $\langle 3e, e \rangle$
(or, equivalently, just $\langle 3, 1 \rangle$)

$|\vec{\nabla} f(1,1)| = |\langle 3e, e \rangle| = \sqrt{9e^2 + e^2} = \sqrt{10e^2} = \sqrt{10}\, e$

$\Rightarrow$ Maximum rate of change is $\sqrt{10}\, e$