MATH 2110Q Exam 1: Things to Know

$|\langle a_1, a_2, a_3\rangle| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

The unit vector in the same direction as $\vec{a}$ is $\dfrac{\vec{a}}{|\vec{a}|}$

$\langle a_1, a_2, a_3\rangle \cdot \langle b_1, b_2, b_3\rangle = a_1b_1 + a_2b_2 + a_3b_3$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

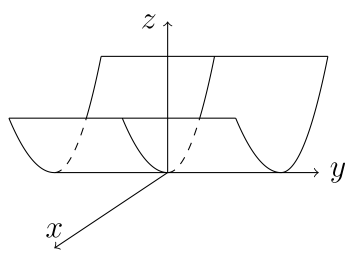
$\vec{a} \cdot \vec{b} = 0 \Rightarrow$ the vectors are orthogonal (perpendicular)

$\vec{a} = \lambda\vec{b}$ (λ is a constant) $\Rightarrow$ the vectors are parallel

$$\begin{aligned}\langle a_1, a_2, a_3\rangle \times \langle b_1, b_2, b_3\rangle &= \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1\rangle \\ &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}\end{aligned}$$

$\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$

For graphing in $\mathbb{R}^3$, if an equation has an absent variable, the surface extends parallel to the axis of the absent variable. For example, $z = x^2$ (below) extends parallel to the y-axis.



We can study and identify quadric surfaces (cones, paraboloids, ellipsoids, hyperbolic paraboloids, hyperboloids) via their traces—cross-sections obtained by fixing one variable.

We should be able to recognize and sketch the following types of traces: parabolas, ellipses, hyperbolas, lines, points.

The line containing (x_0, y_0, z_0) with direction vector $\langle a, b, c\rangle$:
vector equation $\vec{r}(t) = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$
parametric equations $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$

The plane containing (x_0, y_0, z_0) with normal vector $\langle a, b, c\rangle$ has equation
$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

The tangent plane approximation (linearization) of $f(x, y)$ at (a, b) is
$z = f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b)$

The gradient of $f(x, y)$ is $\vec{\nabla} f(x, y) = \langle f_x, f_y\rangle$

The directional derivative of f in the direction of the unit vector $\vec{u}$ is
$D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u}$

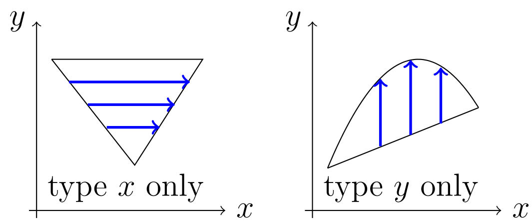
At any point, the maximum derivative of f occurs in the direction $\vec{\nabla} f$, and the value of the maximum derivative is $|\vec{\nabla} f|$

A critical point for $f(x, y)$ is a point that satisfies the system of equations
$f_x = 0$
$f_y = 0$

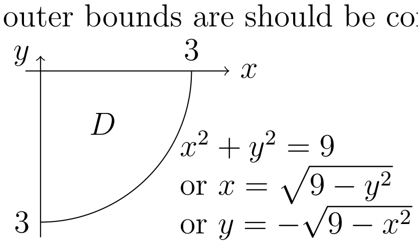
$D = f_{xx} f_{yy} - (f_{xy})^2$. Second Derivative Test on a critical point (a, b):
If $D(a, b) > 0$ and $f_{xx} > 0$, it's a local minimum
If $D(a, b) > 0$ and $f_{xx} < 0$, it's a local maximum
If $D(a, b) < 0$, it's a saddle point
If $D = 0$, test is inconclusive

A region in the xy-plane is type x if the arrows across the region in the direction of the x-axis all start on a single curve and all end on a single curve.

A region in the xy-plane is type y if the arrows across the region in the direction of the y-axis all start on a single curve and all end on a single curve.



To set up a Cartesian double integral (or switch the order of a double integral),
sketch the region being integrated over in the xy-plane
decide what variable to integrate with respect to first
for dy (dx) inside, find equations for the bottom (left) and top (right) bounding curves
outer bounds are should be constant



$$\iint_D f(x,y)\,dA = \int_0^3 \int_{-\sqrt{9-x^2}}^0 f(x,y)\,dy\,dx$$
$$= \int_{-3}^0 \int_0^{\sqrt{9-y^2}} f(x,y)\,dy\,dx$$

$$\iint_D f(x,y)\,dA = \text{Area of } D$$

To convert between polar and Cartesian coordinates:
$x = r\cos\theta$
$y = r\sin\theta$
$x^2 + y^2 = r^2$

In polar coordinates, $dA = r\,dr\,d\theta$