Exam 1 Practice Problems

These questions are not meant to be an exhaustive review and will not provide sufficient practice for the exam, but they should help you begin studying and to diagnose material that you need to practice most.

1. True or False? The line $\vec{r}(t) = \langle 1+t, 1-t, 3t \rangle$ intersects the plane $x+2y-z = 11$ at the point $(-1, 3, -6)$.

2. True or False? Let $f(x, y) = e^x \cos(xy)$. We know that $f(0, \pi) = 1$, but say we wish to use a tangent plane (linearization) to approximate $f(.1, \pi - .1)$. Using the tangent plane at $(0, \pi)$ would yield

$$f(.1, \pi - .1) \approx 1 - \pi(.1).$$

3. True or False? The iterated integral below represents the area of the region D in the first quadrant bounded by $y = \sqrt{3}x$, $y = \frac{1}{\sqrt{3}}x$, and $x^2 + y^2 = 4$.

$$\int_{\pi/6}^{\pi/3} \int_0^2 1 \, dr \, d\theta$$

4. Let $\vec{a} = \langle 5, -1, 2 \rangle$ and $\vec{b} = \langle 3, 1, 1 \rangle$. Find $2\vec{a} - 3\vec{b}$, $\vec{a} \cdot \vec{b}$, and $\vec{a} \times \vec{b}$. Are $\vec{a}$ and $\vec{b}$ parallel? Orthogonal? Why or why not?

5. If the angle between two planes is defined as the angle between their normal vectors, find the cosine of the angle between the planes $x + y = 2$ and $x + y + \sqrt{2}z = \sqrt{6}$.

6. Find and classify all critical points for the function $f(x, y) = \frac{1}{2}y^2 - \frac{1}{3}x^3 - xy + 2x + 5$.

7. Determine a vector equation for <u>any</u> one line that is parallel to the plane $3x - 5y + z = 10$.

8. Let $f(x, y) = x^2(y^3 + 1)^2 + 3x$. Find an equation of the tangent plane at the point $(1, 1, 7)$.

9. Let D be the region in the xy-plane enclosed by $y = x$, $y = -x$, and $x^2 + y^2 = 8$ with $x \geqslant 0$. Sketch the region D and set up a double integral in polar coordinates to compute the area of D. Do NOT evaluate.

10. Consider the double integral $\displaystyle\int_0^5 \int_y^5 e^{x^2} \, dx \, dy$.
    (a) Sketch the region that is being integrated over.
    (b) Evaluate the iterated integral.

11. Give equations and sketches for two different traces of the surface $x^2 + 4y^2 - z^2 = 0$.

12. Let $f(x, y) = x^2 e^{xy}$.
    (a) Find $D_{\vec{u}} f(2, 0)$ if $\vec{u}$ is the unit vector $\left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle$.
    (b) Find the direction in which the derivative of f at $(1, 1)$ is maximized and find the maximum value of the derivative.