

Name: \_\_\_\_\_

Score: \_\_\_\_\_ /20

# Understanding Surfaces

1. (a) Consider the sphere  $x^2 + y^2 + z^2 = 8$ , and the half cone  $z = \sqrt{x^2 + y^2}$  (the full cone is  $z^2 = x^2 + y^2$ , and the other half is  $z = -\sqrt{x^2 + y^2}$ ). Find the intersection of these two surfaces—what kind of curve is it? can we say how large it is?  
(b) Find the equation of a sphere whose intersection with the half cone  $z = \sqrt{x^2 + y^2}$  is a single point.  
(c) Find the equation of a sphere that does not intersect the half cone  $z = \sqrt{x^2 + y^2}$ .
2. For each of the following, describe all possible intersections of the given surfaces. If it is possible for the surfaces to not intersect, be sure to mention this as well.
  - (a) Two planes
  - (b) A plane and a circular cylinder
  - (c) A plane and a sphere
  - (d) Two spheres
  - (e) An elliptic paraboloid and a plane
  - (f) A (true) cone and a plane
3. What is the difference between  $x^2 + y^2 = 9$  and  $x^2 + y^2 \leq 9$ ? What do they look like? Do they both have surface area? volume?
4. Sketch the region in  $\mathbb{R}^3$  given by  $1 \leq x^2 + y^2 \leq 4$ ,  $z \leq 0$ .
5. Assume that gravity acts in the direction of the negative  $z$ -axis. If we poured water from high above the  $xy$ -plane, which of the surfaces  $z = x^2$ ,  $z = y^2$ ,  $x = y^2$ , and  $y = z^2$  would hold the water? Why or why not?