University of Connecticut
Department of Mathematics

MATH 2110Q — PRACTICE EXAM 2 — SPRING 2017

NAME: SOLUTIONS

DISCUSSION SECTION:

Read This First!

- Read the questions and instructions carefully.
- The available points for each problem are given in brackets.
- You must show your work to obtain full credit (and to possibly receive partial credit). **Correct answers with no justification will not receive credit.**
- Make sure your answers are clearly indicated, and cross out any work you do not want graded.
- Do not leave any blanks! Even if you do not arrive at an answer, show as much progress towards a solution as you can, and explain your reasoning.
- Calculators are not allowed.

Grading - For Administrative Use Only

Page:	1	2	3	4	5	Total
Points:	11	11	7	8	13	50
Score:						

1. Compute the arc length of the curve given by $\vec{r}(t) = \langle \frac{4}{3}t^{3/2}, \frac{1}{2}t^2 - 2t, \frac{4}{3}t^{3/2} \rangle$ from $t = 1$ to $t = 2$. [5]

$$\vec{r}\,'(t) = \langle 2\sqrt{t}, t-2, 2\sqrt{t} \rangle$$

$$\Rightarrow |\vec{r}\,'(t)| = \sqrt{4t + (t-2)^2 + 4t} = \sqrt{t^2 + 4t + 4} = t + 2$$

$$\therefore L = \int_a^b |\vec{r}\,'(t)|\,dt = \int_1^2 (t+2)\,dt$$

$$= \left(\frac{1}{2}t^2 + 2t\right)\Big|_1^2 = (2+4) - \left(\frac{1}{2} + 2\right) = \underline{\frac{7}{2}}.$$

2. Compute the line integral of $\vec{F} = \langle y - x, x^3 \rangle$ over the portion of the curve $y = x^2$ from $(-1, 1)$ to $(1, 1)$. [6]

not conservative since $\frac{\partial P}{\partial y} = 1 \neq 3x^2 = \frac{\partial Q}{\partial x}$!

$$C: \vec{r}(t) = \langle t, t^2 \rangle, \quad -1 \le t \le 1$$

$$\Rightarrow \vec{r}\,'(t) = \langle 1, 2t \rangle$$

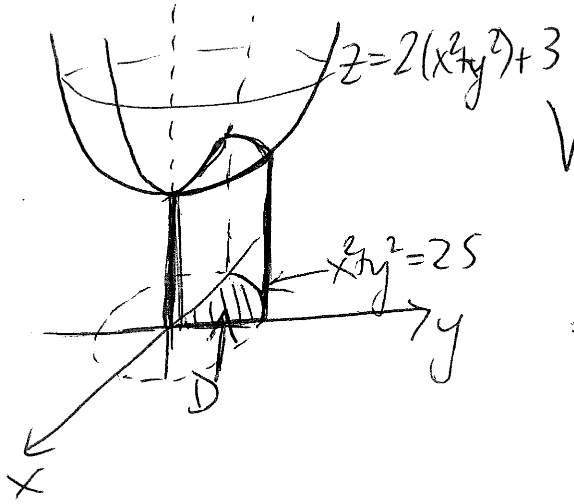
$$\therefore \int_C \vec{F} \cdot d\vec{r} = \int_{-1}^{1} \langle t^2 - t, t^3 \rangle \cdot \langle 1, 2t \rangle\,dt$$

$$= \int_{-1}^{1} (\underset{\text{even}}{t^2} - \underset{\text{odd}}{t} + \underset{\text{even}}{2t^4})\,dt = 2\int_0^1 (t^2 + 2t^4)\,dt$$

(−a to a)

$$= 2\left(\frac{1}{3}t^3 + \frac{2}{5}t^5\right)\Big|_0^1 = 2\left(\frac{1}{3} + \frac{2}{5}\right) = \underline{\frac{22}{15}}.$$

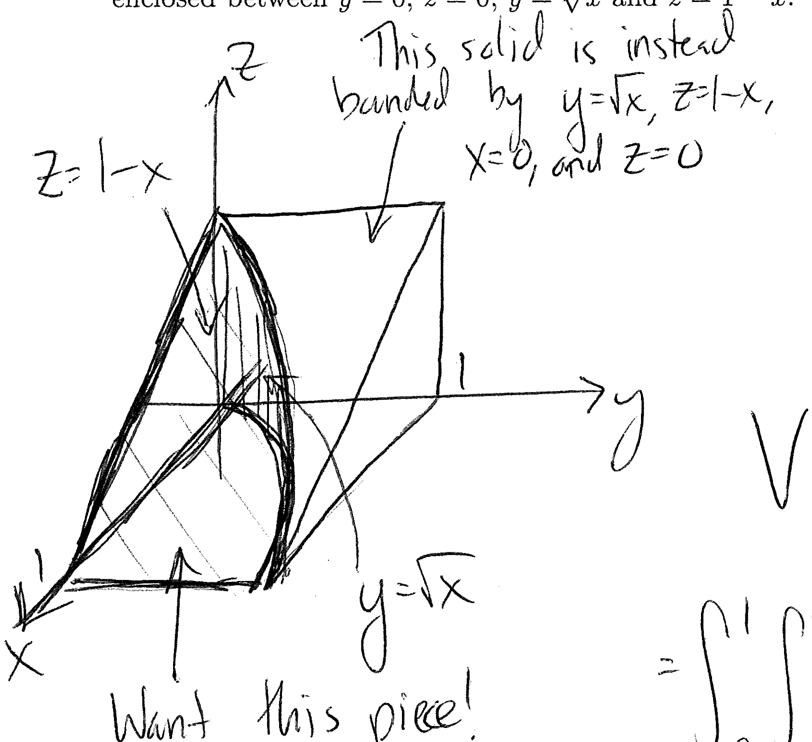
3. Set up but **do not evaluate** a triple integral **in cylindrical coordinates** to find the volume enclosed between the xy-plane and $z = 2(x^2 + y^2) + 3$ over the region D in the *second quadrant* enclosed by $x^2 + y^2 = 25$ using cylindrical coordinates. [5]

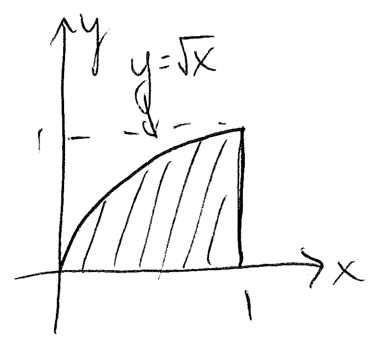


$$V = \iiint_E 1\, dV$$

$$= \int_{\frac{\pi}{2}}^{\pi} \int_0^5 \int_0^{2r^2+3} r \, dz\, dr\, d\theta .$$

4. Set up but **do not evaluate** a triple integral **in Cartesian coordinates** to find the volume enclosed between $y = 0$, $z = 0$, $y = \sqrt{x}$ and $z = 1 - x$. [6]





$$V = \iiint_E 1\, dV$$

$$= \int_0^1 \int_0^{\sqrt{x}} \int_0^{1-x} 1\, dz\, dy\, dx .$$

5. Compute the line integral of $f(x, y) = 4xy$ over the line segment from $(1, -2)$ to $(3, 0)$. [7]

C: $\vec{r}(t) = \langle 1, -2 \rangle + t \langle 3-1, 0-(-2) \rangle$

$= \langle 1+2t, -2+2t \rangle, \quad 0 \le t \le 1$

$\Rightarrow \vec{r}\,'(t) = \langle 2, 2 \rangle$ and $|\vec{r}\,'(t)| = \sqrt{8}$.

$$\therefore \int_C f(x,y)\,ds = \int_0^1 4\underbrace{(1+2t)}_{=x}\underbrace{(-2+2t)}_{=y}\underbrace{\sqrt{8}\,dt}_{=ds}$$

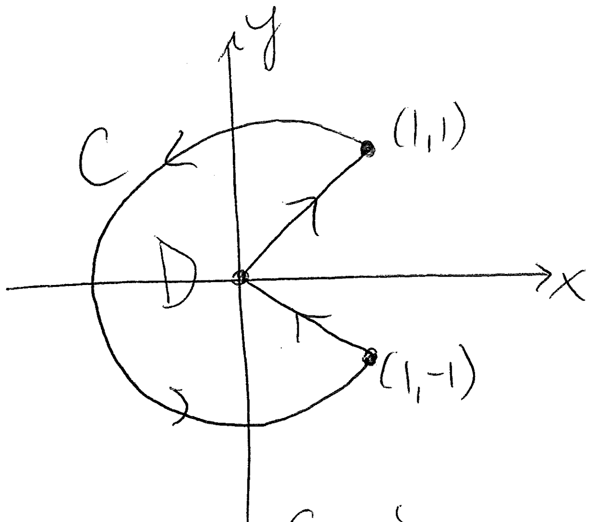
$$= 8\sqrt{8}\int_0^1 (1+2t)(t-1)\,dt$$

$$= 8\sqrt{8}\int_0^1 (2t^2 - t - 1)\,dt$$

$$= 8\sqrt{8}\left(\frac{2}{3}t^3 - \frac{1}{2}t^2 - t\right)\Big|_0^1$$

$$= 8\sqrt{8}\left(\frac{2}{3} - \frac{1}{2} - 1\right) = 8\sqrt{8}\left(-\frac{5}{6}\right) = \frac{-20\sqrt{8}}{3}.$$

6. Let C be the path consisting of the line segment from $(0,0)$ to $(1,1)$, followed by the portion of the circle of radius $\sqrt{2}$ traced counterclockwise from $(1,1)$ to $(1,-1)$, followed by the line segment from $(1,-1)$ back to $(0,0)$. Sketch C, then use Green's Theorem to compute the line integral of $\vec{F} = \langle x^2 - xy, e^{\cos y} \rangle$ over C. [8]



$$\vec{F} = \langle P, Q \rangle = \langle x^2 - xy, e^{\cos y} \rangle$$

$$\Rightarrow \frac{\partial P}{\partial y} = -x \text{ and } \frac{\partial Q}{\partial x} = 0$$

$$\text{so } \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = x.$$

$$\oint_C \vec{F} \cdot d\vec{r} \overset{\substack{\text{Green's} \\ \text{Theorem}}}{=} \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$= \iint_D x \, dA \quad \left(\begin{array}{c} D \text{ is nicely} \\ \text{described in polar} \end{array} \right)$$

$$= \int_{\frac{\pi}{4}}^{\frac{7\pi}{4}} \int_0^{\sqrt{2}} \underset{= x}{(r\cos\theta)} \, \underbrace{r \, dr \, d\theta}_{= dA}$$

$$= \underbrace{\left(\int_{\frac{\pi}{4}}^{\frac{7\pi}{4}} \cos\theta \, d\theta \right)}_{= \frac{-2}{\sqrt{2}}} \underbrace{\left(\int_0^{\sqrt{2}} r^2 \, dr \right)}_{= \frac{2\sqrt{2}}{3}} = \underline{\frac{-4}{3}}.$$

7. Let $\vec{F} = \langle 3x^2 - 2xy + 5, y^3 - x^2 \rangle$ be a vector field.

(a) Find a potential function f so that $\vec{F} = \vec{\nabla} f$. [3]

$$f(x,y) = \int (3x^2 - 2xy + 5)\,dx = x^3 - x^2 y + 5x + g(y)$$

$$\Rightarrow f_y = -x^2 + g'(y) \overset{\text{given}}{=} y^3 - x^2 \Rightarrow g'(y) = y^3 \Rightarrow g(y) = \tfrac{1}{4}y^4 + K \text{ (constant)}$$

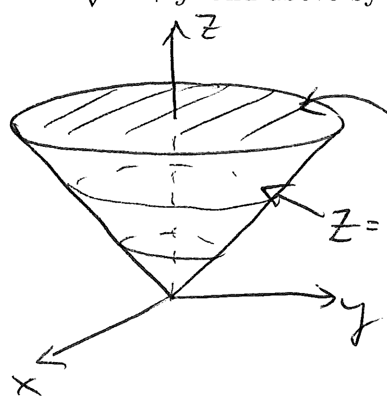
$$\therefore\ f(x,y) = x^3 - x^2 y + 5x + \tfrac{1}{4}y^4 + K.$$

(b) If C is the circle $(x-2)^2 + (y+4)^2 = 9$ traced once clockwise, find the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$. [2]

$\vec{F}$ is conservative and the curve C is closed, so by the Fundamental Theorem for Line ~~Integrals~~ Integrals (or by Green's Theorem) we have $\oint_C \vec{F} \cdot d\vec{r} = 0$.

8. Write the following integral using **spherical coordinates** if the solid E is bounded below by $z = \sqrt{x^2+y^2}$ and above by $z = 1$. **Do not evaluate.** [8]

$$\iiint_E y^2 z\, dV$$



$z = 1 \Rightarrow \rho\cos\phi = 1 \Rightarrow \rho = \sec\phi$

$z = \sqrt{x^2+y^2} \Rightarrow \phi = \frac{\pi}{4}$

$$\therefore \iiint_E y^2 z\, dV = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sec\phi} (\underset{= y}{\rho\sin\phi\sin\theta})^2 (\underset{= z}{\rho\cos\phi}) \underbrace{(\rho^2\sin\phi)\, d\rho\, d\phi\, d\theta}_{= dV}$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sec\phi} \rho^5 \sin^3\phi\cos\phi\sin^2\theta\, d\rho\, d\phi\, d\theta$$