1. Which of the following systems have a unique solution ?

$$(a)\left\{\begin{array}{rcrcr} x & + & y & = & 3 \\ -3x & + & -3y & = & -9 \end{array}\right\}; \quad (b)\left\{\begin{array}{rcrcr} 2x & + & 2y & = & 6 \\ x & + & -3y & = & 7 \end{array}\right\}$$

$$(c)\left\{\begin{array}{rcrcr} x & - & y & = & 4 \\ -x & + & 2y & = & 8 \end{array}\right\}; \quad (d)\left\{\begin{array}{rcrcr} x & + & y & = & 3 \\ x & + & y & = & 6 \end{array}\right\}$$

Solution: There are a variety of ways to solve this, by graphing each set we can see,

(a) Infinite solutions

(b) unique solution

(c) unique solution

(d) no solutions

2. Consider the following system:

$$\begin{array}{rcrcrl} x_1 & + & x_2 & + & x_3 & = 0 \\ x_1 & - & 2x_2 & + & 2x_3 & = 4 \\ x_1 & + & 2x_2 & - & x_3 & = 2 \end{array}$$

Find the solution set.

Solution:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 1 & 2 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & 1 & 4 \\ 0 & 1 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & -3 & 1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & -5 & 10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$x_1 = 4$$
$$x_2 = -2$$
$$x_3 = -2$$

3. Suppose you have a cookie shop. You make two kinds of cookies, a sugar cookie and a chocolate chip cookie. Each sugar cookie takes 2 minutes to bake and each chocolate chip cookie takes 3 minutes to bake. Additionally, each sugar cookie takes 3 grams of sugar and each chocolate chip cookie takes 2 grams of sugar. Suppose you sell your sugar cookies for 1 dollar and each chocolate chip cookie sells for 2 dollars. Suppose you have only 30 minutes to bake cookies and 24 grams of sugar. Find the feasible region and optimize your revenue.

Solution:

Let, x be the number of sugar cookies, and y the number of chocolate chip cookies.

You want to graph the following inequalities

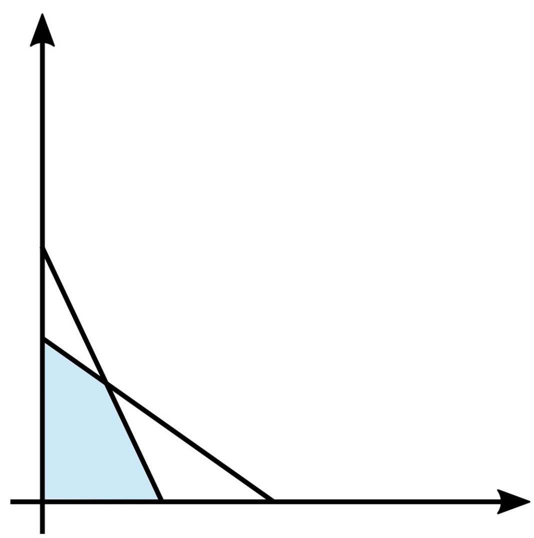
$$2x + 3y \leq 30$$

$$3x + 2y \leq 24$$

$$x \geq 0$$

$$y \geq 0$$

The feasible region will look something like this:



where the corners are: $(0, 0)$, $(0, 10)$, $(8, 0)$, $(2.4, 8.4)$

The revenue function is

$$R(x, y) = x + 2y$$

The revenue function at those points is:

$$R(0, 0) = 0,$$

$$R(0, 10) = 20,$$

$$R(8,0) = 8,$$

$$R(2.4, 8.4) = 19.2$$

So you want to make all chocolate chip cookies to optimize your revenue.

4. Compute the following:

(a)

$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -3 & 4 \\ 4 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 1 \\ 3 & -5 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 3 & -5 \\ 4 & 2 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 1 & 3 & 4 \\ 3 & -5 & 4 \end{bmatrix}^\top + 2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$

Solution:

(a)

$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = 6$$

(b)

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -3 & 4 \\ 4 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 12 \\ 2 & 22 & 4 \\ 13 & -2 & 26 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 1 \\ 3 & -5 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix} = \text{undefined}$$

(d)

$$\begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 3 & -5 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 21 & 5 \\ 6 & 30 \\ 15 & -5 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 1 & 3 & 4 \\ 3 & -5 & 4 \end{bmatrix}^\top + 2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & -1 \\ 6 & 6 \end{bmatrix}$$