1. Which of the following systems have a unique solution ?

$$(a)\left\{\begin{array}{rcrcr} x & + & y & = & 3 \\ -3x & + & -3y & = & -9 \end{array}\right\}; \quad (b)\left\{\begin{array}{rcrcr} 2x & + & 2y & = & 6 \\ x & + & -3y & = & 7 \end{array}\right\}$$

$$(c)\left\{\begin{array}{rcrcr} x & - & y & = & 4 \\ -x & + & 2y & = & 8 \end{array}\right\}; \quad (d)\left\{\begin{array}{rcrcr} x & + & y & = & 3 \\ x & + & y & = & 6 \end{array}\right\}$$

2. Consider the following system:

$$\begin{array}{rcrcrl} x_1 & + & x_2 & + & x_3 & = 0 \\ x_1 & - & 2x_2 & + & 2x_3 & = 4 \\ x_1 & + & 2x_2 & - & x_3 & = 2 \end{array}$$

   Find the solution set.

3. Suppose you have a cookie shop. You make two kinds of cookies, a sugar cookie and a chocolate chip cookie. Each sugar cookie takes 2 minutes to bake and each chocolate chip cookie takes 3 minutes to bake. Additionally, each sugar cookie takes 3 grams of sugar and each chocolate chip cookie takes 2 grams of sugar. Suppose you sell your sugar cookies for 1 dollar and each chocolate chip cookie sells for 2 dollars. Suppose you have only 30 minutes to bake cookies and 24 grams of sugar. Find the feasible region and optimize your revenue.

4. Compute the following:

   (a)
$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

   (b)
$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -3 & 4 \\ 4 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

   (c)
$$\begin{bmatrix} 1 & 1 \\ 3 & -5 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

   (d)
$$\begin{bmatrix} 2 & 1 & 4 \\ 2 & -4 & 4 \\ 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 3 & -5 \\ 4 & 2 \end{bmatrix}$$

   (e)
$$\begin{bmatrix} 1 & 3 & 4 \\ 3 & -5 & 4 \end{bmatrix}^{\top} + 2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$