University of Connecticut
Department of Mathematics

SOLUTIONS

MATH 1070 SAMPLE EXAM 1

Exam 1 will cover sections 4.1-4.7 and 5.1-5.4. This sample exam is intended to be used as one of several resources to help you prepare. The coverage of topics is not exhaustive, and you should look through all examples from lectures, quizzes, and homework as these will all be relevant. The wealth of problems in our text is also a good resource for practice with this material.

- The exam is a closed notes, closed book exam. You can not receive aid on this exam from anyone. Approved calculators are allowed, but there is <u>no sharing of calculators!</u>
- Some partial credit may be given depending on the correctness of the work submitted. You must show all work and calculations needed to reach your answers. Just using a calculator is not sufficient for credit.
- Please make sure to attend the exam that you signed up for at the beginning of the term. The room for your exam can be found on the common course webpage.

1. In a survey of 200 people, it was found that 190 own a car or a bike, while 145 own a car, and 85 own a bike.

   (a) Represent this information in a Venn diagram.

$n(U) = 200$

$n(C \cup B) = 190$

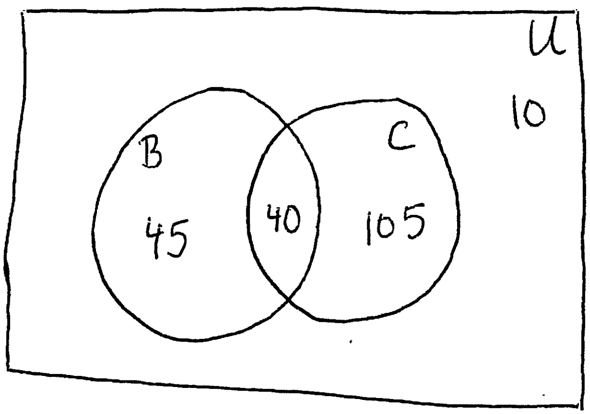
$n(C) = 145$

$n(B) = 85$

$n(C \cap B) = n(C) + n(B) - n(C \cup B)$

$= 148 + 85 - 190$

$= 40$



   (b) If someone is selected randomly from this group of people, what is the probability that she or he owns a bike but not a car? [3]

want $P(B \cap C^c) = \dfrac{n(B \cap C^c)}{n(U)} = \dfrac{45}{200} = \dfrac{9}{40}$

2. We toss a fair six-sided die twice, and record the number that is showing on each toss.

   (a) How many outcomes are there in the sample space?

   6 options for each toss so

   $6 \cdot 6 = 36$ outcomes

   (b) What is the probability that the die shows 5 at least once?

   $n(\text{one } 5) = 2 \cdot 5 = 10$ $\quad P(\text{at least one } 5) = \dfrac{11}{36}$

   $n(\text{two } 5s) = 1$

   (c) What is the probability that the die does not show 5?

   $n(\text{no } 5s) = 5 \cdot 5 = 25$ $\quad P(\text{no } 5s) = \dfrac{25}{36}$

   OR

   $P(\text{no } 5s) = 1 - P(\text{at least one } 5) = 1 - \dfrac{11}{36} = \dfrac{25}{36}$

3. A family is taking a photo of their 3 dogs and 2 cats arranged in a row.

   (a) How many ways are there to arrange the 5 pets in a row?

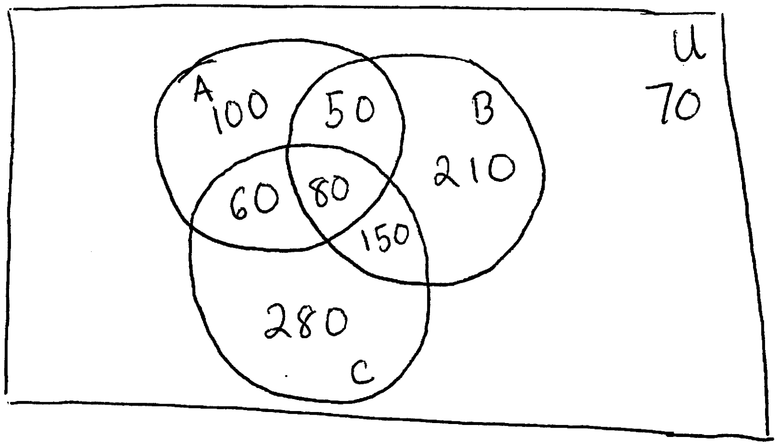
   $P(5,5) = 5!$

   (b) How many ways are there to arrange the 5 pets in a row with if the 2 cats must be next to each other?

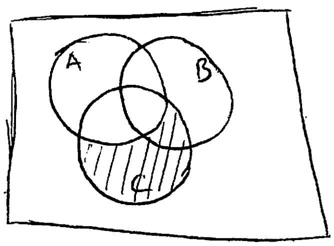
   $2 \cdot 4!$

4. We are given the following information: $n(U) = 1000$, $n(A) = 290$, $n(B) = 490$, $n(C) = 570$, $n(A \cap B) = 130$, $n(A \cap C) = 140$, $n(B \cap C) = 230$, and $n(A \cap B \cap C) = 80$.

   (a) Represent this information in a Venn diagram.

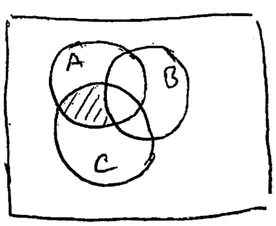


   (b) Find $n(A^c \cap C)$.



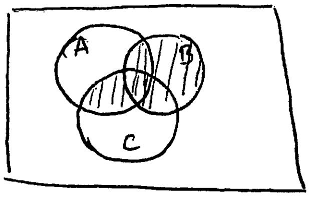
$n(A^c \cap C) = 280 + 150 = 430$

   (c) Find $n(A \cap B^c \cap C)$.



$n(A \cap B^c \cap C) = 60$

   (d) Find $n((A \cap C) \cup B)$.



$n((A \cap C) \cup B) = 60 + 80 + 50 + 150 + 210$
$= 550$

5. In a contest with 20 participants, there will be one 1st prize, two identical 2nd prizes, four identical 3rd prizes, and three identical wild card prizes. The 1st, 2nd and 3rd prizes must all go to different people. The wild card prizes can go to anyone, even those that have won a 1st, 2nd or 3rd prize. How many different ways are there to distribute the prizes?

$$\underset{1st}{\underline{20}} \quad \overbrace{\underset{2nd}{\underline{\quad}}\ \underset{2nd}{\underline{\quad}}}^{C(19,2)} \quad \overbrace{\underset{3rd}{\underline{\quad}}\ \underset{3rd}{\underline{\quad}}\ \underset{3rd}{\underline{\quad}}\ \underset{3rd}{\underline{\quad}}}^{C(17,4)} \qquad \overbrace{\underset{WC}{\underline{\quad}}\ \underset{WC}{\underline{\quad}}\ \underset{WC}{\underline{\quad}}}^{C(20,3)}$$

$$20 \cdot C(19,2) \cdot C(17,4) \cdot C(20,3)$$

6. Let the sample space be $S = \{a, b, c, d\}$. How many possible events are there? Explain your answer.

The number of events is the number of subsets.
Lets count the number of subsets with 0, 1, 2, 3 and 4 elements, and then add them up.

$$C(4,0) + C(4,1) + C(4,2) + C(4,3) + C(4,4)$$

7. Suppose we draw a 5-card hand from a standard 52-card deck.

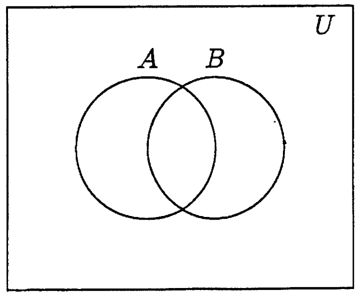
   (a) How many different hands contain a pair of sevens, a different pair, and one card of a different value, e.g. two sevens, two kings, and one ten?

   $C(4,2) \cdot 12 \cdot C(4,2) \cdot 11 \cdot 4$

   (b) How many different hands contain three cards of one suit, and a pair of cards of a different suit, e.g. three diamonds and two spades?

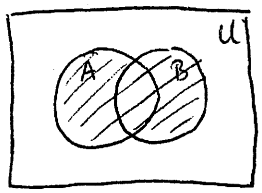
   $4 \cdot C(13,3) \cdot 3 \cdot C(13,2)$

8. Using the following diagram shade the sets indicated. (You may wish to recopy this diagram for each set).

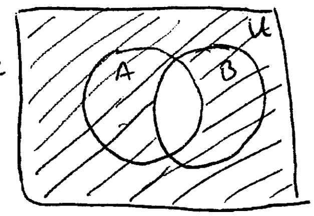


A. $A \cup B$
B. $A^c \cup B^c$
C. $A^c \cap B^c$
D. $(A \cup B)^c$
E. $U \cap A^c$
F. $U \cup B$
G. $A^c \cup B^c \cup (A \cap B)$
H. $A \cap A^c$

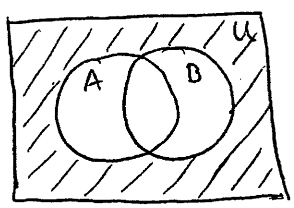
A. $A \cup B$



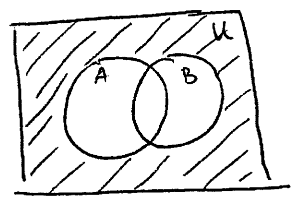
B. $A^c \cup B^c$



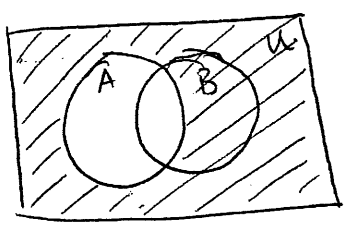
C. $A^c \cap B^c$



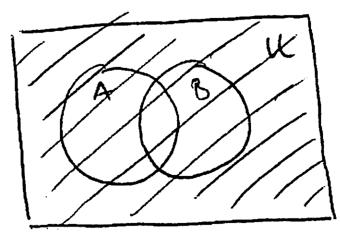
D. $(A \cup B)^c$



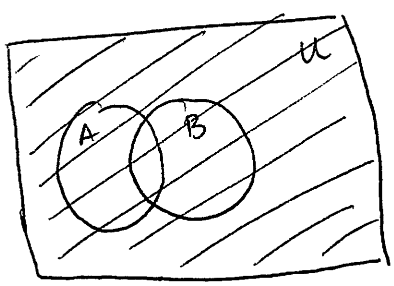
E. $U \cap A^c$



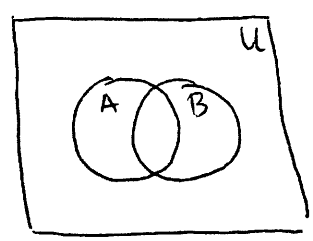
F. $U \cup B$



G. $A^c \cup B^c \cup (A \cap B)$

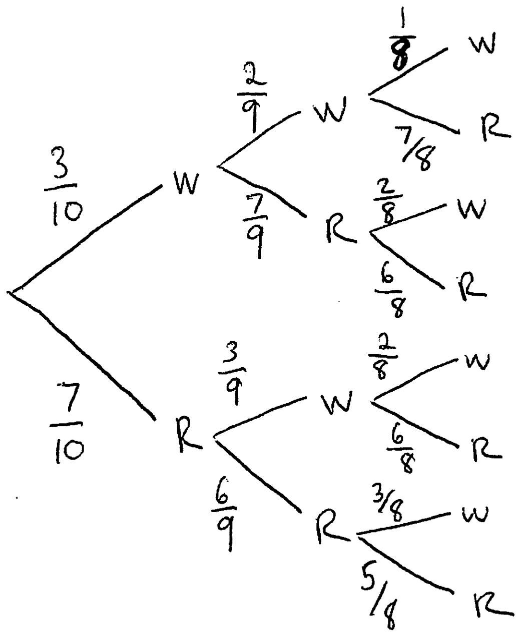


H. $A \cap A^c$



9. Three balls are randomly drawn (without replacement) from an urn that contains three white and seven red balls.

   (a) Draw a tree diagram and indicate the correct probabilities.



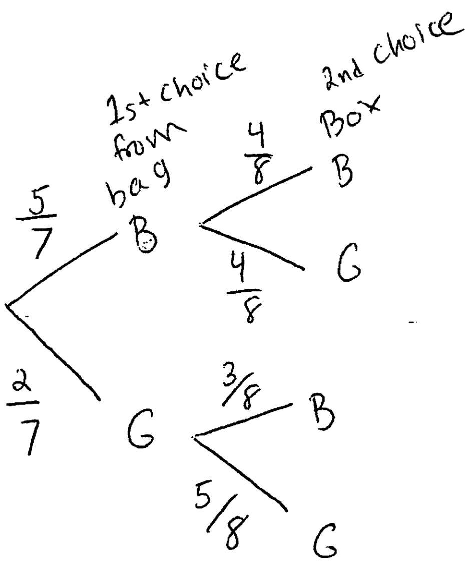
   (b) What is the probability of drawing a white ball on the third draw?

$$\left(\frac{3}{10}\cdot\frac{2}{9}\cdot\frac{1}{8}\right)+\left(\frac{3}{10}\cdot\frac{7}{9}\cdot\frac{2}{8}\right)+\left(\frac{7}{10}\cdot\frac{3}{9}\cdot\frac{2}{8}\right)+\left(\frac{7}{10}\cdot\frac{6}{9}\cdot\frac{3}{8}\right)$$

   (c) What is the probability of drawing a white ball on the third draw given that at least one white ball was drawn on the first two draws?

$$\frac{\left(\frac{3}{10}\cdot\frac{2}{9}\cdot\frac{1}{8}\right)+\left(\frac{3}{10}\cdot\frac{7}{9}\cdot\frac{2}{8}\right)+\left(\frac{7}{10}\cdot\frac{3}{9}\cdot\frac{2}{8}\right)}{\frac{3}{10}+\frac{7}{10}\cdot\frac{3}{9}}$$

10. A bag contains five blue and two green jelly beans. A box contains three blue and four green jelly beans. A jelly bean is selected at random from the bag and is placed in the box. Then a jelly bean is selected at random from the box. If a green jelly bean is selected from the box, what is the probability that the transferred jelly bean was blue?



$$P\left(\text{1st choice blue} \;\middle|\; \text{2nd choice green}\right)$$

$$= \frac{\frac{5}{7}\cdot\frac{4}{8}}{\frac{5}{7}\cdot\frac{4}{8} + \frac{2}{7}\cdot\frac{5}{8}}$$

11. A basketball player makes on average 3 free throws out of every 5 attempted. If the player attempts 7 free throws, find the probability that they make at least five of them.

$$P(FT) = \frac{3}{5}$$

$$P(\text{at least 5 made out of 7}) = P(5) + P(6) + P(7)$$

$$= C(7,5)\left(\tfrac{3}{5}\right)^5\left(\tfrac{2}{5}\right)^2 + C(7,6)\left(\tfrac{3}{5}\right)^6\left(\tfrac{2}{5}\right)^1 + C(7,7)\left(\tfrac{3}{5}\right)^7\left(\tfrac{2}{5}\right)^0$$

12. A baseball player has a batting average of 0.250 (this is the probability of getting a hit each time they bat). The player bats 4 times in a game.

(a) What is the probability that the player gets exactly 2 hits?

$$P(2\text{ hits}) = C(4,2)\ (.250)^2\ (.750)^2$$

(b) What is the probability that the player gets 4 hits given that they had at least 2 hits?

$$P(4\text{ hits} \mid \text{at least 2 hits}) = \frac{P(4\text{ hits})}{P(\text{at least 2 hits})} = \frac{P(4\text{ hits})}{P(2\text{hits}) + P(3\text{hits}) + P(\ldots}$$

$$= \frac{C(4,4)\ (.250)^4\ (.750)^0}{C(4,2)(.250)^2(.750)^2 + C(4,3)(.250)^3(.750)^1 + C(4,4)(.250^4)\ (.75\ldots}$$