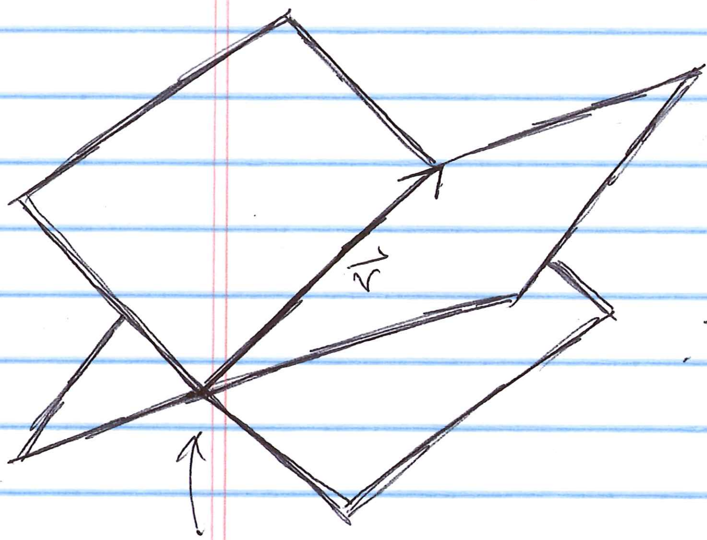
§12.5 Example Relating Lines and Planes

Q: Find an equation for the line of intersection for the planes $x+y-z=1$ and $x-3y+4z=1$.



direction vector $\vec{v}$ is in both planes, i.e., it is normal/orthogonal to the normal vectors of both planes!

A: $x+y-z=1 \Rightarrow \vec{n}_1 = \langle 1,1,-1 \rangle$

$x-3y+4z=1 \Rightarrow \vec{n}_2 = \langle 1,-3,4 \rangle$

$$\therefore \vec{v} = \vec{n}_1 \times \vec{n}_2$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \langle 4-3, -(4+1), -3-1 \rangle$$

$$= \langle 1, -5, -4 \rangle$$

Therefore, if we know a point on the line like $(1,0,0)$ (check that it lies in both planes!), an equation of the line of intersection is

$$\vec{r}(t) = \langle 1,0,0 \rangle + t\langle 1,-5,-4 \rangle$$

$$\Rightarrow \underline{\vec{r}(t) = \langle 1+t, -5t, -4t \rangle}$$