§12.5 Example Relating Lines and Planes

Q: Find an equation for the line of intersection for the planes \( x + y - z = 1 \) and \( x - 3y + 4z = 1 \).

A: \( x + y - z = 1 \Rightarrow \vec{n}_1 = \langle 1, 1, -1 \rangle \)

\( x - 3y + 4z = 1 \Rightarrow \vec{n}_2 = \langle 1, -3, 4 \rangle \)

\[ \vec{v} = \vec{n}_1 \times \vec{n}_2 \]

\[ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -3 & 4 \end{vmatrix} \]

\[ = \langle 4 - 3, -(4 + 1), -3 - 1 \rangle \]

\[ = \langle 1, -5, -4 \rangle \]

direction vector \( \vec{v} \)
is in both planes, i.e., it is normal/orthogonal to the normal vectors of both planes.

Therefore, if we know a point on the line, like \( (1, 0, 0) \) (check that it lies in both planes), an equation of the line of intersection is

\[ \vec{r}(t) = \langle 1, 0, 0 \rangle + t \langle 1, -5, -4 \rangle \]

\[ \Rightarrow \vec{r}(t) = \langle 1 + t, -5t, -4t \rangle \]