

Name: _____

Score: _____ /20

Review for Final Exam

Questions 1-10 are true/false. Explain your reasoning in either case, and make corrections to make the statement true if it is false.

1. For any two vectors \vec{u} and \vec{v} , then $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$.
2. The line $\vec{r}(t) = \langle 1 + 3t, -t, 2 + 2t \rangle$ is parallel to the plane $x + y - z = 4$.
3. For $f(x, y) = \ln(x^2y^3)$, $f_{xy} = f_{yx} = 0$.
4. If $f(x, y)$ is differentiable and $\vec{\nabla}f(a, b) = \vec{0}$, then (a, b) is a local maximum, local minimum, or saddle.

5.

$$\int_0^1 \int_{-1}^1 \int_{xy}^{x+y} f(x, y, z) dz dy dx = \int_{-1}^1 \int_0^1 \int_{xy}^{x+y} f(x, y, z) dz dx dy$$

6. The surface $z = r^2$ is a cone in cylindrical coordinates.

7. The iterated integral

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^2 \rho \sin^2 \phi d\rho d\theta d\phi$$

represents the volume of the portion of the sphere of radius 2 in the first octant.

8. If $\vec{F} = \vec{\nabla}f$ for some function f , then for any curve C we have

$$\int_C \vec{F} \cdot d\vec{r} = 0.$$

9. If $-C$ denotes the curve C traced with opposite orientation, then

$$\int_{-C} f(x, y) ds = \int_C f(x, y) ds.$$

10. If \vec{F} is a conservative vector field and S is a surface with closed boundary curve C , then

$$\iint_S \text{curl} \vec{F} \cdot d\vec{S} = 0.$$

11. Find an equation of the line through the point $(3, -2, 8)$ that is orthogonal to the plane $z = x + y$.

12. Determine and describe the intersection of the surfaces $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 = 1$.

13. Let $f(x, y) = \sqrt{9 - x^2 - y^2}$.

- (a) Sketch the domain of f .
- (b) Describe the traces of f .
- (c) Sketch the graph of f . What type of surface is this?

14. Marine biologists have determined that when a shark detects blood in the water, it will swim in the direction in which the concentration of blood increases the fastest. If the concentration at any point is approximated by $C(x, y) = e^{1(x^2+2y^2)}$, in which direction will the shark move at the point $(1, 1)$?

15. Find an equation of the tangent plane to the surface $z = x \sin(x + y)$ at the point $(-1, 1, 0)$.

16. Find and classify all critical points for the function $f(x, y) = x^4 + y^4 - 4xy + 1$.

17. Evaluate the iterated integral

$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3) dy dx.$$

18. Let R be the region in the first quadrant enclosed by $x^2 + y^2 = 4$, $x = 0$, and $y = x$. Evaluate the double integral

$$\iint_R 3xy dA.$$

19. Let E be the region contained between $x^2 + y^2 + z^2 = 8$ and $z = \sqrt{x^2 + y^2}$. Set up but do NOT evaluate the following integral in Cartesian, cylindrical, and spherical coordinates:

$$\iiint_E xz dV.$$

20. If a particle's position is given by the curve $\vec{s}(t) = \langle \sqrt{5}t, e^t, e^{-t} \rangle$, find the velocity and acceleration at $t = 0$.

21. Find an equation of the tangent line at the point $(0, -2, 1/2)$ for the curve $\vec{r}(t) = \left\langle t \ln t, -\sqrt{3t+1}, \frac{e^{t-1}}{1+t^2} \right\rangle$.

22. Find the length of the curve C given by $\vec{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t \rangle$, $0 \leq t \leq 1/3$.

23. Which of the following integrals makes sense? Explain why or why not.

$$\int_C \vec{F} \cdot d\vec{r} \quad \int_C \text{curl} \vec{F} ds \quad \int_C \vec{\nabla} \cdot \vec{F} ds \quad \int_C \text{div} \vec{F} \cdot d\vec{r} \quad \int_C \vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) ds$$

24. Assuming $\vec{F} = \langle x, y, z \rangle$ and C is the unit circle $x^2 + y^2 = 1$, $z = 0$, traversed counterclockwise, compute any integral that made sense in the previous question.

25. Let $\vec{F}(x, y, z) = \langle 2xy^2, z^3 + 2x^2y, 3yz^2 \rangle$.

(a) Find $\text{curl} \vec{F}$.

(b) Find $\text{div} \vec{F}$.

(c) Is \vec{F} conservative? Explain your answer.

(d) Can you find a function $f(x, y, z)$ so that $\vec{\nabla} f = \vec{F}$? Explain your answer.

(e) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ if C is given by

$$\vec{r}(t) = \left\langle t^2 e^{\cos(\pi t/4)}, t e^{t-2} \cos\left(\frac{\pi}{2}t\right), e^{t-2} \sin\left(\frac{\pi}{2}t\right) \right\rangle, 0 \leq t \leq 2.$$

26. If C is the line segment from $(0, 0, 0)$ to $(1, 2, 3)$, evaluate

$$\int_C z e^y ds.$$

27. Determine an example of a non-constant vector field \vec{F} that has both $\text{curl} \vec{F} = \vec{0}$ AND $\text{div} \vec{F} = 0$.

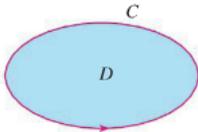
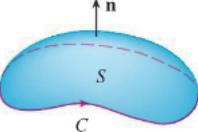
28. Compute $\iint_S \text{curl} \vec{F} \cdot d\vec{S}$ if $\vec{F} = \langle z, y, x \rangle$ and S is given by $x^2 + y^2 = 1$, $0 \leq z \leq 4$ and $x^2 + y^2 \leq 1$, $z = 4$.

29. Compute $\iint_S \text{curl} \vec{F} \cdot d\vec{S}$ if $\vec{F} = \langle x^2 - y^2z, xy^3 + z, z^3 - (x^2 + y^2) \rangle$ if S is the surface given by $z^2 = x^2 + y^2$, $0 \leq z \leq 3$ and $x^2 + y^2 \leq 9$, $z = 3$.

Hint: Think about what the surface looks like. Which theorem(s) could be applied?

30. If $\vec{F}(x, y, z) = \langle x + z, y + z, z \rangle$ and S is the surface given by $x^2 + y^2 + z^2 = 1$ with $z \geq 0$ with upward orientation, compute BOTH of the surface integrals

$$\iint_S \vec{F} \cdot d\vec{S} \quad \text{and} \quad \iint_S \text{curl} \vec{F} \cdot d\vec{S}.$$

Fundamental Theorem of Calculus	$\int_a^b F'(x) dx = F(b) - F(a)$	
Fundamental Theorem for Line Integrals	$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$	
Green's Theorem	$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_C P dx + Q dy$	
Stokes' Theorem	$\iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$	
Divergence Theorem	$\iiint_E \text{div} \mathbf{F} dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$	