University of Connecticut
Department of Mathematics

MATH 2110Q | PRACTICE EXAM 1 | FALL 2016

NAME: Solutions

DISCUSSION SECTION: ______________________

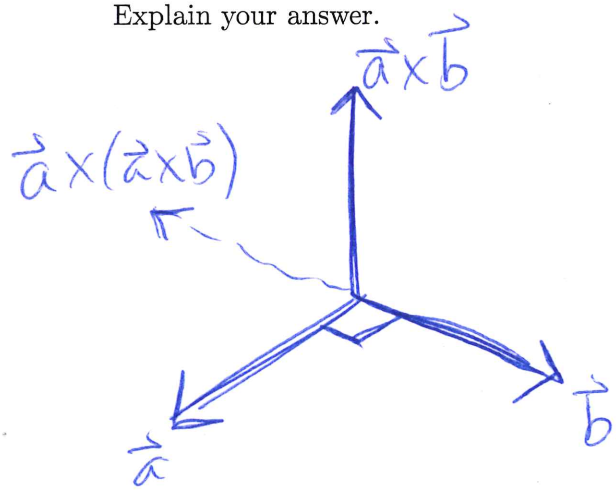
Read This First!

- Read the questions and instructions carefully.
- The available points for each problem are given in brackets.
- You must **show your work** to obtain full credit (and to possibly receive partial credit). Correct answers with no justification will not receive credit.
- Make sure your answers are clearly indicated, and cross out any work you do not want graded.
- Do not leave any blanks! Even if you do not arrive at an answer, show as much progress towards a solution as you can, and explain your reasoning.
- Calculators are allowed, but models that can do symbolic computations (TI-89 and above, including TI-NSpire) are not allowed.

Grading - For Administrative Use Only

Page:	1	2	3	4	5	Total
Points:	11	11	9	10	9	50
Score:						

1. Assume that $\vec{a}$ and $\vec{b}$ are orthogonal, nonzero vectors. In what direction does $\vec{a} \times (\vec{a} \times \vec{b})$ point? Explain your answer. [5]



Since $\vec{a}$ and $\vec{b}$ are orthogonal, the angle between any pair of $\vec{a}$, $\vec{b}$, and $\vec{a} \times \vec{b}$ is $\frac{\pi}{2}$. 2

If we take $\vec{a} \times (\vec{a} \times \vec{b})$ and apply the Right-hand Rule, we can see that the vector will point in the $\boxed{-\vec{b} \text{ direction.}}$ 3

2. If the angle between two planes is defined as the angle between their normal vectors, find the angle between the planes $x + y = 2$ and $x + y + \sqrt{2}z = \sqrt{6}$. [6]

$x+y=2 \Rightarrow \vec{n}_1 = \langle 1, 1, 0 \rangle$ 1

$x+y+\sqrt{2}z = \sqrt{6} \Rightarrow \vec{n}_2 = \langle 1, 1, \sqrt{2} \rangle$ 1

$$\underbrace{\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}}_{1} = \frac{1+1+0}{\sqrt{1+1+0}\sqrt{1+1+2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \quad 2$$

$\therefore\ \theta = \arccos\left(\frac{1}{\sqrt{2}}\right)$ with $0 \le \theta \le \pi$,

so $\boxed{\theta = \frac{\pi}{4}.}$ 1

3. Find an equation for the line containing the point $(-3, 4, 2)$ that is parallel to the line $\vec{r}(t) = \langle 1-t, 2+3t, 5t \rangle$. [5]

The direction of $\vec{r}(t)$ is $\vec{v} = \langle -1, 3, 5 \rangle$, and a parallel line will have the same direction. 2

Therefore, the parallel line through $(-3,4,2)$ is

$$\boxed{\vec{r}(t) = \langle -3, 4, 2 \rangle + t\langle -1, 3, 5 \rangle = \langle -3-t, 4+3t, 2+5t \rangle}$$

2 1

4. The total resistance R produced by three conductors with resistances R_1, R_2, and R_3 connected in a parallel electric circuit is given by the formula [6]

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \quad \Rightarrow R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

Find $\partial R / \partial R_1$.

$$\Downarrow$$

$$R^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1}$$

$$\therefore \quad \underbrace{-R^{-2} \cdot \frac{\partial R}{\partial R_1}}_{2} = \underbrace{-R_1^{-2}}_{2}$$

$$\Rightarrow \boxed{\begin{aligned} \frac{\partial R}{\partial R_1} &= R^2 \cdot R_1^{-2} \\ &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-2} R_1^{-2} \\ &= \frac{R_1^{-2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^2} \end{aligned}}$$

2

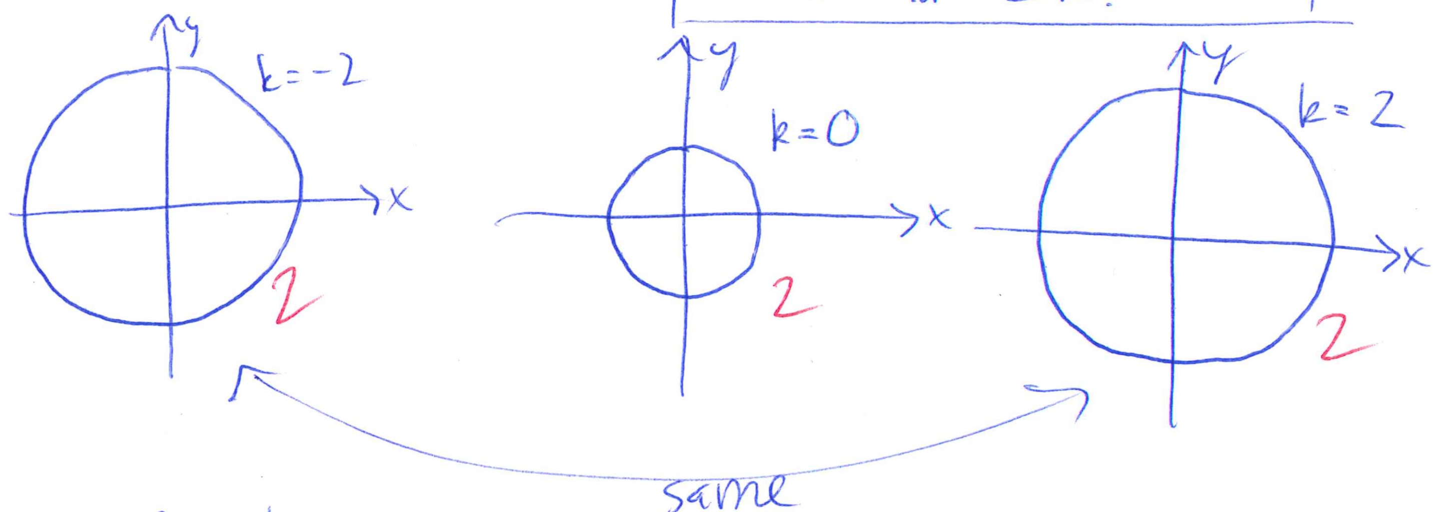
(several ways to write answer)

5. Describe the shape of the traces $z = k$ for the surface $x^2 + y^2 - z^2 = 1$. Sketch a graph of a trace for at least one value each with $k < 0$, $k = 0$, and $k > 0$. [9]

$$z = k \Rightarrow x^2 + y^2 - k^2 = 1 \Rightarrow \underbrace{x^2 + y^2 = 1 + k^2}$$

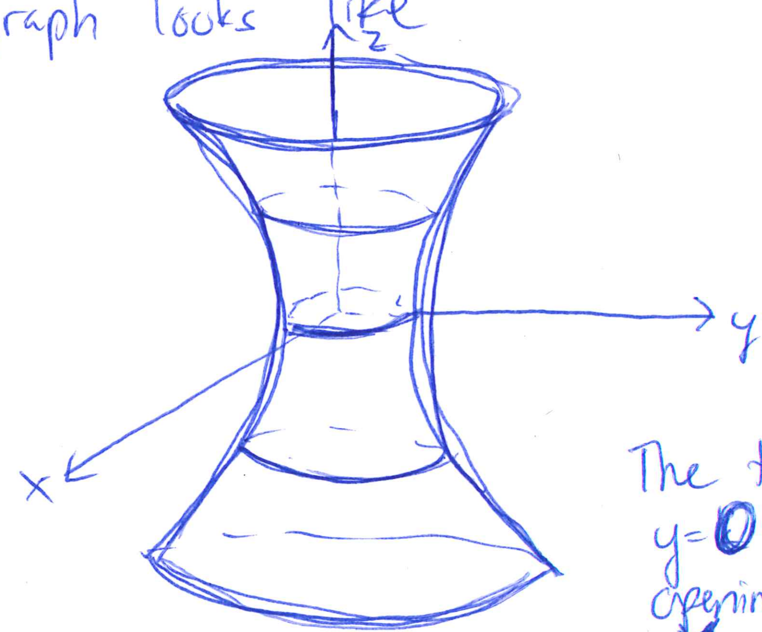
3

$\boxed{\text{circles centered at } (0,0) \text{ with radius } \sqrt{1+k^2}, \text{ same for } \pm k.}$



For your information:

This surface is called a <u>one-sheet hyperboloid</u>, and it's graph looks like



The traces $x = 0$ and $y = 0$ are hyperbolas opening in the ~~x~~-direction or y-direction, respectively. ($x = 0 \Rightarrow$ opens y-direction)

6. Consider the function $f(x,y) = x^2(y^3+1)^2 + 3x$.

(a) Compute $\vec{\nabla} f$. [5]

$f_x = 2x(y^3+1)^2 + 3$ 2

$f_y = 2x^2(y^3+1) \cdot 3y^2 = 6x^2y^2(y^3+1)$ 2

$\therefore \boxed{\vec{\nabla} f = \underbrace{\langle f_x, f_y \rangle}_{1} = \langle 2x(y^3+1)^2+3,\ 6x^2y^2(y^3+1) \rangle}$

(b) Find an equation of the tangent plane at the point $(1,1,7)$. [5]

$f_x(1,1) = 2(1+1)^2 + 3 = 11$ 1

$f_y(1,1) = 6 \cdot 1 \cdot 1(1+1) = 12$ 1

$\Rightarrow z = z_0 + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$ 1

$\Rightarrow \boxed{z = 7 + 11(x-1) + 12(y-1)}$

$= 7 + 11x - 11 + 12y - 12$

$\therefore \boxed{z = 11x + 12y - 16}$

either is fine 1

7. Determine a function $f(x, y)$ that has partial derivatives $f_x = x + 4y$ and $f_y = 3x - y$. If no such function exists, explain why not. [5]

Two approaches

1) Using Clairaut's Theorem, we know $f_{xy} = f_{yx}$. However, $f_{xy} = 4$ and $f_{yx} = 3$, so no such function $f(x,y)$ exists. 2 for no, 3 for explanation

2) If $f_x = x+4y$, then $f(x,y)$ would contain a term of $4xy$. That means that f_y would have a $4x$ term, but it does not, so no such function exists.

8. If $f(x, y) = 2x^2 - 3y$, find $D_{\vec{u}} f(1, 1)$ if $\vec{u}$ is the unit vector $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$. [4]

$\vec{\nabla} f = \langle 4x, -3 \rangle \Rightarrow \vec{\nabla} f(1,1) = \langle 4, -3 \rangle$ 1

$\therefore D_{\vec{u}} f(1,1) = \vec{\nabla} f(1,1) \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ 2

$= \langle 4, -3 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

$= \frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}}$

$= \boxed{\frac{1}{\sqrt{2}}}$ 1