Math1071Q Fall 2015
Exam 2 review sheet

The first question is to give your practice with differentiation. Some of these questions may be answered using techniques covered for Exam 1, but differentiation is also an essential skill for Exam 2.

1. Find $\frac{d}{dx}(f(x))$ where

i $f(x) = \sqrt{2}$

ii $f(x) = x^{1.4}$

iii $f(x) = 4 - x - x^2$

iv $f(x) = \frac{1}{3}e^x$

v $f(x) = 4 - 3\ln x$

vi $f(x) = x^5 - 5e^x - 1$

vii $f(x) = \frac{x^2-3x-6}{3x}$

viii $f(x) = 3x^2 + \frac{3}{x^2}$

ix $f(x) = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

x $f(x) = \frac{x+\frac{3}{x}}{\sqrt{x}}$

xi $f(x) = \sqrt{x^3}$

xii $f(x) = x^3e^x$

xiii $f(x) = x^4\ln x$

xiv $f(x) = (e^x+1)(\sqrt{x}+1)$

xv $f(x) = \left(e^x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right)$

xvi $f(x) = \frac{x}{x+2}$

xvii $f(x) = \frac{x+1}{x^2+2}$

xviii $f(x) = \frac{e^x}{x-2}$

xix $f(x) = \frac{-4\ln x}{x^4+3}$

xx $f(x) = e^{3x}$ (without using the chain rule)

xxi $f(x) = e^{3x}$ (this time using the chain rule)

xxii $f(x) = (2x+1)^{15}$

xxiii $f(x) = \sqrt{e^x}$

xxiv $f(x) = \frac{3}{e^x+1}$

xxv $f(x) = \frac{1}{\sqrt{\ln x}}$

xxvi $f(x) = e^x(x^2+1)^8$

xxvii $f(x) = (5x+3)^3(x^2-4)^6$

xxviii $f(x) = \sqrt{\sqrt{x}+1}$

xxix $f(x) = \frac{e^x+1}{(2x+3)^3}$

xxx $f(x) = e^{\sqrt[3]{x}}$

xxxi $f(x) = \ln|x^3+x^2+1|$

xxxii $f(x) = \ln\left|\frac{x}{x+1}\right|$

xxxiii $f(x) = e^{-x^2}\ln(x^2)$

2. Sketch the graph of $f(x)$.

   i $f(x) = x^4 - 2x^3 + x^2$

   ii $f(x) = \frac{x+1}{x-1}$

   iii $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

3. i Suppose the price and demand of a commodity is related by $p(x) = e^{-2x}$. Find the marginal revenue $R(x)$ and find where the marginal revenue is zero.

   ii If the cost function is given by $C(x) = \ln(9x^2 + 5) + 100$, find the marginal cost. Find where the marginal cost is positive.

   iii If the demand equation for a firm is given by $p = -0.2x + 16$, find the value of x at which the revenue is maximal.

   iv Let the cost function for a firm be given by $C(x) = 5 + 8x$ and let the demand equation be the same as that in the previous question. Find the value at which the profit is maximal.

4. i Find $\lim_{x\to\infty} \frac{x^2}{x^3+1}$

   ii Find $\lim_{x\to\infty} \sqrt{x}$

   iii Find $\lim_{x\to\infty} \frac{x^4 - x^2 + x - 1}{x^3+1}$

   iv Find $\lim_{x\to\infty} \frac{x^2-1}{\sqrt{x}-3}$

   v Find $\lim_{x\to\infty} \sqrt{4 - \frac{1}{x}}$

   vi Find $\lim_{x\to\infty} \sqrt{x^2+1} - x$

5. i Let $f(x) = -2x^3 + 3x^2$. Locate the absolute maximum and absolute minimum values of f on the intervals $[-2, 0]$, $[0, 2]$ and $[-2, 2]$.

   ii Let $f(x) = 3 - x + x^2$. Locate the absolute maximum and absolute minimum values (if they exist) of f on the interval $(-\infty, \infty)$.

   iii Let $f(x) = 3x + x^{-3}$. Locate the abolute maximum and absolute minimum values (if they exist) of f on the interval $(0, 3]$.

   iv If the revenue function for a firm is given by $R(x) = -x^3 + 75x$, then find where the revenue is maximised.

   v What are the dimensions of the rectangular field of $20,000$ square feet that will minimise the cost of fencing if one side costs three times as much per unit length as the other three?

   vi A hotel has 10 luxury units that it will rent out during the peak season at \$300 per day. From experience management knows that one unit will become vacant for each \$50 increase in charge per day. What rent should be charged to maximize revenue?

vii Find the most economical proportions for a closed cylindrical can that will hold 16 cubic inches.

6. i Use a linear approximation to approximate $\sqrt{25.5}$.

   ii Use a linear approximation to approximate $(1.8)^3$.

   iii If the side of a square decreases from 4 inches to 3.8 inches, use the linear approximation formula to estimate the change in area.

7. i If $x = \frac{1}{10+p}$, then find the elasticity E at the points $p = 5$, $p = 10$ and $p = 100$ and determine whether the demand is elastic, inelastic or of unit elasticity.

   ii If $x = \frac{1}{p^3}$, then find the value (if any) of x such that $E = 1$ and the values (if any) of x for which revenue is maximised.