Math 1071Q, Exam I review

Q1 a) $(-\infty, \infty)$

b) $x+1 \geq 0$ so $x \geq -1$ ie. $[-1, \infty)$

c) $x^2 - 1 = 0$ when $x^2 = 1$ ie. when $x = \pm 1$

so $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

d) $x - 2 > 0$ so $x > 2$ so $(2, \infty)$.

e) $2x + 3 > 0$ ie. $2x > -3$ ie. $x > -3/2$

so $(-3/2, \infty)$.

Q2 a) $\frac{1}{3^{-x}} \cdot 9^{x+1} = 1$

ie. $3^x \cdot (3^2)^{x+1} = 1$

i.e. $3^x \cdot 3^{2x+2} = 1$

ie. $3^{3x+2} = 3^0$

so $3x + 2 = 0$ and so $x = -2/3$

b) $4^{2x} = 8^{9x+15}$

ie. $4^{2x} = (2^2)^{2x} = 8^{9x+15} = (2^3)^{9x+15}$

ie. $2^{4x} = 2^{27x+45}$

so $4x = 27x + 45$

or $-23x = 45$ so $x = -45/23$

c) $5^{2x-1} \cdot 5 = 1/5^x$

ie. $5^{2x} = 5^{-x}$

so $2x = -x$

or $3x = 0$

so $x = 0$

d) $2\log_{10}(x+7) + 3 = 0$

ie. $\log_{10}(x+7) = -3/2$

so $x+7 = 10^{-3/2}$

or $x = 10^{-3/2} - 7$

e) $7 \cdot 3^{2x+4} - 1 = 0$

so $3^{2x+4} = 1/7$

so $2x+4 = \log_3 1/7$

and so $x = \dfrac{\log_3 1/7 - 4}{2}$

f) $\log_2(2x-2) - \log_2(x-1) = \log_2 1$

so $\log_2\left(\dfrac{2x-2}{x-1}\right) = \log_2 1$

ie. $\dfrac{2x-2}{x-1} = 1$

so $2x-2 = x-1$

or $x = 1$

g) $\ln(x^2+2) - \ln(3x) = 0$

so $\ln\left(\frac{x^2+2}{3x}\right) = 0$

so $\frac{x^2+2}{3x} = e^0 = 1$

ie. $x^2 + 2 = 3x$

so $x^2 - 3x + 2 = 0$

ie. $(x-1)(x-2) = 0$

so $x = 1$ or $x = 2$.

Q3

$r = 0.036$; $m = 4$

a) $F = 18000$, $t = 3/2$

so $P = \frac{F}{(1+r/m)^{mt}} = \frac{18000}{(1+\frac{0.036}{4})^6}$

$= 17057.897784\ldots$

$\simeq 17057.90$

b) $r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$

$= \left(1 + \frac{0.036}{4}\right)^4 - 1$

$= 0.0364889\ldots$

So approximately 3.649%.

Q4 $C(x) = 2x + 10$; $R(x) = -2x^2 + 20x$

a) $R(x) = -2x^2 + 20x = ax^2 + bx + c$

so $a = -2$, $b = 20$ and $c = 0$.

Revenue is maximised when $x = -\frac{b}{2a} = -\frac{20}{-4} = 5$.

The maximal revenue is $R(5) = -2(5)^2 + 20(5)$
$= 50$.

b) $P(x) = R(x) - C(x)$
$= -2x^2 + 20x - (2x + 10)$
$= -2x^2 + 18x - 10$

so $P(x) = 0$ when $2x^2 - 18x + 10 = 0$

ie when $x = \frac{18 \pm \sqrt{18^2 - 4(2)(10)}}{4} = \frac{18 \pm \sqrt{244}}{4}$

$= \frac{9 \pm \sqrt{61}}{2}$

so the break-even quantities are $x \simeq 0.59488$
& $x \simeq 8.4051$.

Q5 The graph of the depreciation curve is a line containing the two points $(0, 1800)$ and $(11, 1382)$.
This has slope $m = \dfrac{1382 - 1800}{11 - 0} = -\dfrac{418}{11} = -38$.

So if $p(t)$ is the price at time t, then
$$p - 1800 = -38(t - 0)$$
ie. $p = -38t + 1800$.

Q6 a) $C(x) = 55x + 18500$
$R(x) = 190x$
so $P(x) = R(x) - C(x) = 190x - (55x + 18500)$
$= 135x - 18500$.

b) $P(x) = 0$ when $135x - 18500 = 0$
ie. when $x = \dfrac{18500}{135} = \dfrac{3700}{27} \approx 137$.

c) $R\left(\dfrac{3700}{27}\right) = 190\left(\dfrac{3700}{27}\right) = \dfrac{703000}{27}$
$= 26037.037037\ldots$
≈ 26037.04

Q7 $r = 0.1$ and $F = Pe^{+rt}$.
We want to find when $F = 2P$
ie. when $2P = Pe^{+rt}$
or $2P = Pe^{+0.1t}$
so $2 = e^{+0.1t}$
ie. $\ln(2) = +0.1t$ so $t = 10\ln(2)$.

Q8 a) $\lim_{x\to 2} \frac{x^2+x-6}{x^2-8x+12} = \lim_{x\to 2} \frac{(x+3)(x-2)}{(x-2)(x-6)}$

$= \lim_{x\to 2} \frac{x+3}{x-6} = -\frac{5}{4}$

b) $\lim_{x\to 2} \frac{x^2+x-6}{x^2-8x} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x(x-8)}$

$= \frac{0}{-12} = 0$

c) $\lim_{x\to 2} \frac{x^2+x}{x^2-8x+12} = \lim_{x\to 2} \frac{x(x+1)}{(x-2)(x+6)}$

This limit does not exist.

Q9

$$f(x) = \begin{cases} x^2-x+3 & \text{if } x<1 \\ 2 & \text{if } x=1 \\ \dfrac{6x-6}{x^2-1} & \text{if } x>1 \end{cases}$$

a) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{6x-6}{x^2-1} = \lim_{x\to 1^+} \frac{6(x-1)}{(x+1)(x-1)}$

$= \lim_{x\to 1^+} \frac{6}{x+1} = \frac{6}{2} = 3$

b) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2-x+3) = 3.$

c) $\lim_{x\to 1} f(x) = 3.$

d) There is a discontinuity at $x=1$ since
$\lim_{x\to 1} f(x) = 3 \neq f(1) = 2$.

Otherwise, the function is continuous.
So, $f(x)$ is continuous on $(-\infty,1)\cup(1,\infty)$.

Q 10

a) $f(x) = 4x^3 - 2x^2 + 7x + 1$
$[x,a] = [1,4]$.
So $\dfrac{f(x)-f(a)}{x-a} = \dfrac{f(1)-f(4)}{1-4}$
$= \dfrac{4(1)^3 - 2(1)^2 + 7(1) + 1 - (4(4)^3 - 2(4)^2 + 7(4) + 1)}{-3}$
$= \dfrac{4 - 2 + 7 + 1 - (256 - 32 + 28 + 1)}{-3} = \dfrac{-243}{-3} = +81$.

b) 29
c) 21
d) 16.04
e) 15.1004

These are computed in the same way as above.
Note that $f'(x) = 12x^2 - 4x + 7$ & $f'(1) = 15$.

Q11 a) $f(x) = 5x^2 + 7x - 3$

So $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{5(x+h)^2 + 7(x+h) - 3 - 5x^2 - 7x + 3}{h}$

$= \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 + 7x + 7h - 3 - 5x^2 - 7x + 3}{h}$

$= \lim_{h \to 0} \frac{10xh + 5h^2 + 7h}{h} = \lim_{h \to 0} 10x + 5h + 7$

$= 10x + 7.$

So $f'(0) = 7. = m_{tan}$

Moreover, $f(0) = -3.$

So the equation of the tangent line is

$y - (-3) = 7(x - 0)$

ie. $y + 3 = 7x$

so $y = 7x - 3.$

Q 11 b) $f(x) = \dfrac{9}{x-3}$

so $f'(x) = \lim_{h \to 0} \dfrac{1}{h}\left[\dfrac{9}{x+h-3} - \dfrac{9}{x-3}\right]$

$= \lim_{h \to 0} \dfrac{1}{h}\left[\dfrac{9(x-3) - 9(x+h-3)}{(x+h-3)(x-3)}\right]$

$= \lim_{h \to 0} \dfrac{1}{h}\left[\dfrac{9x - 27 - 9x - 9h + 27}{(x+h-3)(x-3)}\right]$

$= \lim_{h \to 0} \dfrac{1}{h}\left[\dfrac{-9h}{(x+h-3)(x-3)}\right]$

$= \lim_{h \to 0} \dfrac{-9}{(x+h-3)(x-3)} = -\dfrac{9}{(x-3)^2}$

So $f'(1) = -\dfrac{9}{(-2)^2} = -\dfrac{9}{4} = m_{tan}$

& $f(1) = \dfrac{9}{-2}$

Thus an equation for the tangent line is

$$y - \left(-\frac{9}{2}\right) = -\frac{9}{4}(x-1)$$

ie $y + 9/2 = -9/4\, x + 9/4$

So $y = -9/4\, x - 9/4$.

Q17 a) $f'(x) = 9x^2 - 20x + 5$

b) $g(x) = 5\ln(x^3) + 7e^x + 7e^3$

$= \ln(x^{15}) + 7e^x + 7e^3$

so $g'(x) = \dfrac{15x^{14}}{x^{15}} + 7e^x = \dfrac{15}{x} + 7e^x$

c) $h'(x) = 1/3 + \dfrac{3}{2\sqrt{x}} + \dfrac{3}{2x^{3/2}} - \dfrac{10}{3x^{5/3}}$.

d) $k(x) = (1 + x + 4x^2) \cdot x^{-1}$

so $k'(x) = \dfrac{d}{dx}(1 + x + 4x^2) \cdot x^{-1} + (1 + x + 4x^2) \cdot \dfrac{d}{dx}(x^{-1})$

$= \dfrac{(1 + 8x)}{x} + (-1)\dfrac{1 + x + 4x^2}{x^2}$

$= \dfrac{x + 8x^2 - 1 - x - 4x^2}{x^2} = \dfrac{4x^2 - 1}{x^2}$