University of Connecticut
Department of Mathematics

MATH 1131Q — SAMPLE FINAL EXAM — SPRING 2017

NAME: Solutions.

DISCUSSION SECTION: ____________________

Read This First!

- This is a sample exam to demonstrate the different type of concepts that could be tested on the actual final exam. This exam does not cover every single concept that has been taught during the semester. Students should use this as a guide to prepare for the final exam.
- Student are encouraged to look at the sample exams from previous midterms to supplement their study plan and strategies to prepare for the final exam.
- Calculators are allowed, but models that can do symbolic computations (TI-89 and above, including TI-NSpire) are not allowed.

Grading - For Administrative Use Only

Page:	1	2	3	4	5	6	7	8	Total
Points:	14	15	13	8	13	13	12	12	100
Score:									

1. Evaluate each of the following limits.

   (a) $\lim\limits_{x \to 3^+} \dfrac{2x^2 - 5x - 3}{x^2 - 6x + 9}$ [4]

$$\lim_{x \to 3^+} \frac{(2x+1)(x-3)}{(x-3)^2} = \lim_{x \to 3^+} \frac{(2x+1)}{(x-3)} \to \infty .$$

@ $x = 3$ the limit DNE

   (b) $\lim\limits_{x \to 0} x^2 \ln x$ [5]

$$\lim_{x \to 0} \frac{\ln x}{1/x^2} \xrightarrow{L'H} \lim_{x \to 0} \frac{1/x}{-2/x^3} \to \lim_{x \to 0} -\frac{x^2}{2} = 0$$

   (c) $\lim\limits_{x \to \infty} \dfrac{x^3 - 2x}{\sqrt{4x^6 + 11}}$ [5]

$$\frac{\lim\limits_{x \to \infty} \frac{x^3}{x^3} - \frac{2x}{x^3}}{\lim\limits_{x \to \infty} \sqrt{\frac{4x^6 + 11}{x^6}}} = \frac{\lim\limits_{x \to \infty} 1 - \frac{2}{x^2}}{\lim\limits_{x \to \infty} \sqrt{4 + \frac{11}{x^6}}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

2. Find the derivative of each of the following functions. **Do not simplify your answers.**

(a) $f(x) = \dfrac{\cos x - \sqrt{x}}{\ln x}$ [5]

$$f'(x) = \frac{\ln(x)\cdot\frac{d}{dx}(\cos x - \sqrt{x}) - (\cos x - \sqrt{x})\cdot\frac{d}{dx}\ln(x)}{(\ln x)^2}$$

$$= \frac{\ln x \cdot \left[-\sin x - \frac{1}{2\sqrt{x}}\right] - (\cos x - \sqrt{x})\cdot\frac{1}{x}}{(\ln x)^2}$$

(b) $f(x) = x^3 e^{\tan x}$ [5]

$$f'(x) = 3x^2 \cdot e^{\tan x} + x^3 \cdot e^{\tan x} \cdot \sec^2 x$$

3. Find the linearization of $f(x) = \sin(2x)$ at $x = \frac{\pi}{6}$. [5]

$$L(x) = f(a) + f'(a)(x-a) \qquad a = \pi/6$$

$$f'(x) = \cos(2x)\cdot 2$$

$$f'(a) = \cos\left(2\cdot\frac{\pi}{6}\right)\cdot 2$$

$$= \cos\left(\frac{\pi}{3}\right)\cdot 2$$

$$= \frac{\sqrt{3}}{2}\cdot 2 = \sqrt{3}$$

$$L(x) = \sin(\pi/3) + \sqrt{3}\left(x - \frac{\pi}{6}\right)$$

$$= \frac{\sqrt{3}}{2} + \sqrt{3}\,(x - \pi/6)$$

4. Given $4e^y \cos x + x^3 = 14 \ln y$, find $\frac{dy}{dx}$ [6]

$4 \cdot e^y \cos x + x^3 = 14 \ln y$

$4 \frac{d}{dx}(e^y \cos x) + (x^3) = \frac{d}{dx} 14 \ln y$

$4 \cdot [e^y \cdot y' \cos x + e^y \cdot (-\sin x)] + 3x^2 = 14 \cdot \frac{1}{y} \cdot y'$

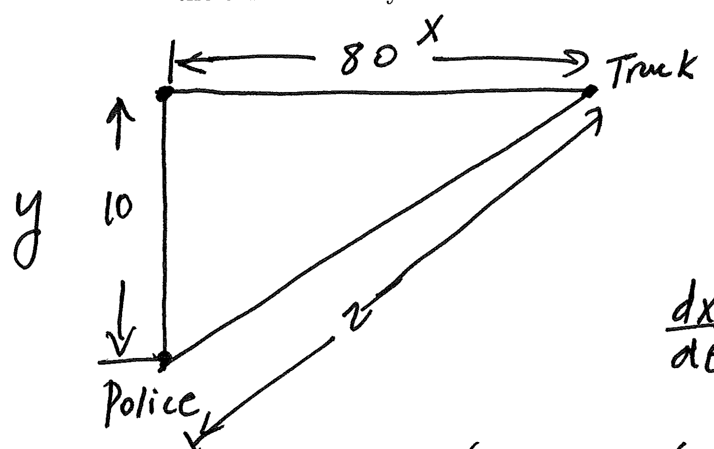
$4e^y y' \cos x \mp 4e^y \sin x + 3x^2 = 14\frac{y'}{y}$

$4e^y \cdot \cos x \cdot y' - 14\frac{y'}{y} = 4e^y \sin x - 3x^2$

$\Rightarrow \left(4e^y \cos x - \frac{14}{y}\right) y' = 4e^y \sin x - 3x^2$

$y' = \frac{4e^y \sin x - 3x^2}{\left(4e^y \cos x - \frac{14}{y}\right)}$

5. A police car is parked 10 ft off a straight road. A truck travels along the road. At the moment when the truck is 80 ft from the point on the road closest to the police car, it is determined that the rate of change of the distance between the truck and the police car is 50 ft/s. Find the truck's velocity at this moment. **Give an exact answer.** [7]



$z^2 = x^2 + y^2$

$z^2 = 80^2 + 10^2 = 6500$

$z = \sqrt{6500}$.

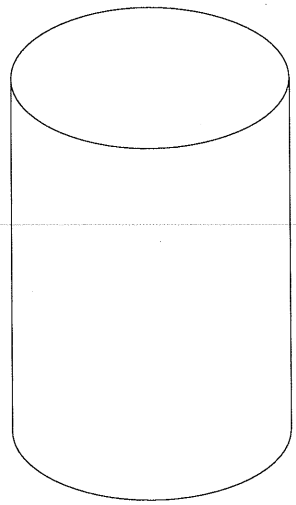
$\frac{dx}{dt} = ?$ $\quad \frac{dz}{dt} = 50$ ft/s $\quad \frac{dy}{dt} = 0$

$\Rightarrow$ police is not moving

$\cancel{2}z\frac{dz}{dt} = \cancel{2}x \cdot \frac{dx}{dt} + \cancel{2y\frac{dy}{dt}}^{\,0}$

$\sqrt{6500} \cdot 50 = 80 \cdot \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \boxed{\sqrt{6500} \cdot \frac{5}{8}}$

6. We are constructing a cylindrical container with **no lid** that must have volume 120 cm^3. Find the radius and height of the container that will minimize the amount of material (i.e. surface area) required. If the container has radius r and height h, the circular base has area πr^2, the sides have area $2\pi rh$, and the volume is $\pi r^2 h$. **Give exact answers.** [8]



$V = \pi r^2 h \qquad h = \frac{120}{\pi r^2}$

$SA = 2\pi r h + \pi r^2$

$SA = \frac{2\cancel{\pi}\cancel{r} \cdot 120}{\cancel{\pi} r^{\cancel{2}}} + \pi r^2$

$s(r) = \frac{240}{r} + \pi r^2$

$\frac{ds}{dr} = \frac{-240}{r^2} + 2\pi r$

$\frac{ds}{dr} = 0 \Rightarrow \frac{240}{r^2} = 2\pi r \qquad r^3 = \frac{240}{2\pi} = \frac{120}{\pi} \quad r = \sqrt[3]{\frac{120}{\pi}}.$

$\frac{d^2s}{dr^2} = \frac{480}{r^3} + 2\pi \quad @ \ r = \sqrt[3]{\frac{120}{\pi}} \quad \frac{480}{\frac{120}{\pi}} + 2\pi = 6\pi > 0$

$\therefore \ r = \sqrt[3]{\frac{120}{\pi}}$ is minimum

$h = \frac{120}{\pi \cdot \left(\frac{120}{\pi}\right)^{2/3}}$

7. Evaluate each of the following integrals. **Do not simplify your answers.**

(a) $\int (6 \sec x \tan x - 3\sqrt{x} + e)\, dx$ [4]

$\int 6 \sec x \tan x - \int 3\sqrt{x} + \int e\, dx$

$6 \cdot \sec x - \frac{3 x^{3/2}}{3/2} + ex + c$

(b) $\int_1^3 \left(e^{3x} - \frac{4}{x^3}\right) dx$ [4]

$\frac{e^{3x}}{3} + \frac{4}{2x^2} \Big|_1^3$

$\left(\frac{e^9}{3} + \frac{2}{9}\right) - \left(\frac{e^3}{3} - \frac{2}{1}\right)$

$\frac{e^9}{3} - \frac{e^3}{3} + \frac{2}{9} - 2 = \frac{e^9 - e^3}{3} - \frac{16}{9}$

8. The function $v(t) = \frac{5}{4t} + 9t^2$ gives the velocity in m/s of a particle at any time $t > 0$. Find a formula for the particle's position function $s(t)$ if we know that $s(1) = 10$. [5]

$s(t) = \int v(t)\, dt = \int \left(\frac{5}{4t} + 9t^2\right) dt = \frac{5}{4} \ln t + 9\frac{t^3}{3} + c$

$s(1) = 10 \Rightarrow \frac{5}{4} \cancel{\ln(1)}^{\,0} + \frac{9}{3} \cdot (1)^2 + c = 10$

$c = 7$

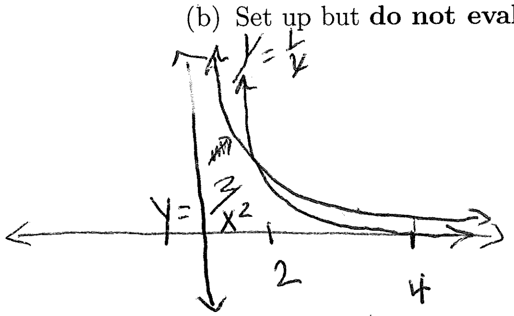
$s(t) = \frac{5}{4} \ln(t) + 3t^3 + 7$

9. Consider the area enclosed by $y = \frac{1}{x}$, $y = \frac{2}{x^2}$, and $x = 4$.

(a) Find the x-coordinate of the point where $y = \frac{1}{x}$ and $y = \frac{2}{x^2}$ intersect. [3]

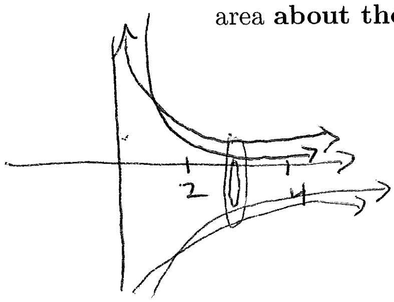
$$\frac{1}{x} = \frac{2}{x^2} \Rightarrow x = 2$$

(b) Set up but **do not evaluate** an integral to compute the area described above. [3]



$$\int_2^4 \left(\frac{1}{x} - \frac{2}{x^2}\right) dx$$

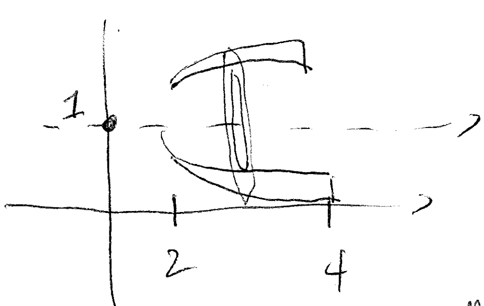
(c) Set up but **do not evaluate** an integral to compute the volume obtained by rotating the area **about the x-axis.** [3]



$$\int_2^4 \left(\left(\frac{1}{x}\right)^2 \pi - \left(\frac{2}{x^2}\right)^2 \pi\right) dx$$

OR $$\pi \int_2^4 \left[\left(\frac{1}{x}\right)^2 - \left(\frac{2}{x^2}\right)^2\right] dx$$

(d) Set up but **do not evaluate** an integral to compute the volume obtained by rotating the area **about the line $y = 1$.** [4]



$$\int_2^4 \left(1 - \frac{1}{x}\right)^2 \pi - \left(1 - \frac{2}{x^2}\right)^2 \pi$$

OR $$\pi \int_2^4 \left(1 - \frac{1}{x}\right)^2 - \left(1 - \frac{2}{x^2}\right)^2$$

10. Evaluate each of the following integrals. **Do not simplify your answers.**

(a) $\int \frac{2x^3+3x}{x^4+3x^2-7}\,dx$ [5]

$u = x^4 + 3x^2 - 7$

$\frac{du}{dx} = 4x^3 + 6x$

$du = 2(2x^3+3x)\,dx$

$\int \frac{du}{2 \cdot u} = \frac{1}{2} \cdot \ln|u| + c$

$= \frac{1}{2} \ln|x^4 + 3x^2 - 7| + c$

(b) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} e^{\cos x} \sin x \, dx$ [7]

$u = \cos x$

$du = -\sin x \, dx$

$x = \pi/3 \quad u = \cos \pi/3 = \frac{1}{2}$

$x = \pi/2 \quad u = \cos \pi/2 = 0$

$\int_{1/2}^{0} -e^u \, du = -\int_{1/2}^{0} e^u \, du$

$= -[e^u]_{1/2}^{0} = -[e^0 - e^{1/2}]$

$= e^{1/2} - 1$

11. Let $A(x) = \int_1^x (2t^2 - 12t + \frac{11}{3})\,dt$. Find the intervals on which the graph $y = A(x)$ is concave up and concave down, and find all points of inflection. [6]

$y' = A'(x) = 2x^2 - 12x + \frac{11}{3}$

$y'' = 4x - 12$

$y'' = 0 \Rightarrow 4x - 12 = 0$

$x = 3.$

$(-\infty, 3)$ concave down

$(3, \infty)$ concave up.

$A(x) = f(x)$ $\cap$ $\cup$

$-\infty$ $x=0$ 3 4 ∞

$f''(x) < 0$ $f''(x) > 0$

12. Circle to indicate whether each statement is true or false, and **justify your answers.**

(a) The value of $\int_{-2}^{1} \frac{5}{x}\,dx$ is $5\ln|x|\Big|_{-2}^{1} = 5\ln 1 - 5\ln 2.$ [3]

True **False**

The function $\frac{5}{x}$ is not defined.

(b) The function $f(x) = \dfrac{x^2+3x-10}{x^2-5x+6}$ has a vertical asymptote at $x = 2$. [3]

True **False**

$\dfrac{(x+5)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)}$ $f(x) = \dfrac{(x+5)}{(x-3)}$ No V.A @ $x = 2$.