University of Connecticut
Department of Mathematics

MATH 1131Q | PRACTICE EXAM 3 | SPRING 2017

NAME: Solution.

DISCUSSION SECTION: Note: please check with your TA or Instructor if you have any concerns or doubts about the soln.

Read This First!

- Read the questions and instructions carefully.
- The available points for each problem are given in brackets.
- You must **show your work** to obtain full credit (and to possibly receive partial credit). Correct answers with no justification will not receive credit.
- Make sure your answers are clearly indicated, and cross out any work you do not want graded.
- Do not leave any blanks! Even if you do not arrive at an answer, show as much progress towards a solution as you can, and explain your reasoning.
- Calculators are allowed, but models that can do symbolic computations (TI-89 and above, including TI-NSpire) are not allowed.

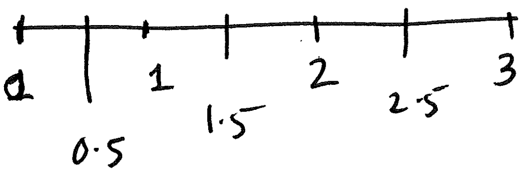
Grading - For Administrative Use Only

Page:	1	2	3	4	Total
Points:	12	15	8	15	50
Score:					

1. Let $f(x) = \sqrt{x} + 3x^2 + 2$

   (a) Approximate the area under the graph $y = f(x)$ over $[0, 3]$ using three subintervals and midpoints (M_3). **Round your answer to two decimal places.** [6]

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{3} = 1.$$



$$[f(0.5) + f(1.5) + f(2.5)]\,\Delta x$$

$$f(x) \approx [3.46 + 9.97 + 22.33]\,1$$

$$\approx 35.76.$$

   (b) Evaluate $\int_0^3 (\sqrt{x} + 3x^2 + 2)\,dx$ to find the exact area under $y = f(x)$ over $[0, 3]$. **Give an exact answer.** [6]

$$\int_0^3 (x^{1/2} + 3x^2 + 2)\,dx$$

$$= \frac{2}{3}x^{3/2} + x^3 + 2x \Big|_0^3$$

$$= \frac{2}{3}(3)^{3/2} + 27 + 6$$

$$= 2\sqrt{3} + 33$$

2. Find the absolute maximum and minimum values of $f(x) = x^2 e^x$ over the interval $[-4, -1]$. [8]
Round your answers to three decimal places.

$f(x) = x^2 e^x$

$f'(x) = 2x \cdot e^x + x^2 \cdot e^x$

$f'(x) = 0 \Rightarrow (2x e^x + x^2 \cdot e^x) = 0$

$= e^x x(2+x) = 0$

$\Rightarrow x = 0$ OR $x = -2$

	x	$f(x)$	
min	-4	$16 \cdot e^{-4} = 16/e^4 \approx 0.293$	
Max	-2	$4 \cdot e^{-2} = \frac{4}{e^2} \approx 0.541$	
do not consider	0	0	Not in the interval
	-1	$1 \cdot e^{-1} = \frac{1}{e} \approx 0.368$	

3. Find the intervals where $f(x) = x^4 - 4x^3 - 18x^2 + 10$ is concave up and concave down, and identify any points of inflection. [7]

$f'(x) = 4x^3 - 12x^2 - 36x$

$f''(x) = 12x^2 - 24x - 36$

$f''(x) = 0 \Rightarrow 12x^2 - 24x - 36 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ OR $x = 3$

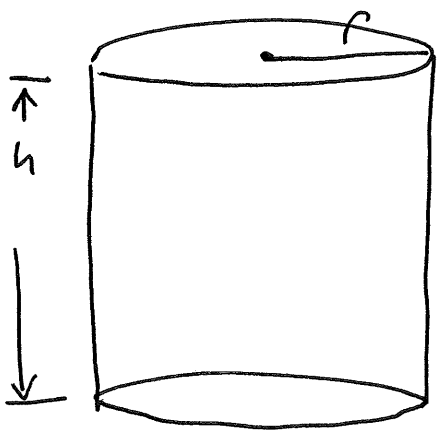
I.P $x = -1$ $x = 3$

$-\infty$, $x=-2$, $x=-1$, $x=0$, $x=3$, $x=4$, ∞

Int	x	$f''(x)$	$f(x)$
$-\infty, -1$	-2	$60 > 0$	up
$-1, 3$	0	$-36 < 0$	down
$3, \infty$	4	$60 > 0$	up

4. We are constructing a cylindrical container with a lid. The surface area of the container must be 12 ft^2. Find the radius and height of the container that will maximize its volume. [8]

Note: If a cylinder has radius r and height h, its total surface area (top, bottom, and sides) is $S = 2\pi r^2 + 2\pi rh$ and its volume is $V = \pi r^2 h$.



$V = \pi r^2 h.$

$S = 2\pi r^2 + 2\pi rh$

$12 = 2\pi r^2 + 2\pi rh$

$= 2\pi r(r+h)$

$\frac{6}{\pi r} = r + h \Rightarrow h = \frac{6}{\pi r} - r$

$V = \pi r^2 \cdot h = \pi r^2\left(\frac{6}{\pi r} - r\right) = 6r - \pi r^3$

$V(r) = 6r - \pi r^3$

$V'(r) = 6 - 3\pi r^2$

$V'(r) = 0 \Rightarrow 6 - 3\pi r^2 = 0$

$3\pi r^2 = 6$

$r^2 = \frac{2}{\pi}$

$r = \sqrt{\frac{2}{\pi}}$

$V''(r) = -6\pi r$

$V''\left(r = \sqrt{\frac{2}{\pi}}\right) = -6\pi \cdot \sqrt{\frac{2}{\pi}} < 0$

$\therefore r = \sqrt{\frac{2}{\pi}}$ is local max by 2^{nd} derivative test

$h = \frac{6}{\pi \cdot \sqrt{\frac{2}{\pi}}} - \sqrt{\frac{2}{\pi}}$

$h = \frac{6}{\sqrt{\pi} \cdot \sqrt{2}} - \frac{\sqrt{2}}{\sqrt{\pi}} = \frac{6-2}{\sqrt{\pi \cdot 2}} = \frac{4}{\sqrt{2\pi}}$

for maximum volume $r = \sqrt{\frac{2}{\pi}}$, $h = \frac{4}{\sqrt{2\pi}}$

5. Evaluate the limit $\lim_{x\to 0} x\ln(x^2)$. $0\cdot\infty$ [7]

$$\lim_{x\to\infty} \frac{\ln(x^2)}{1/x} \quad \frac{\infty}{\infty} \xrightarrow{L'H} \lim_{x\to\infty} \frac{\frac{1}{\cancel{x^2}}\cdot 2x}{\cancel{-1/x^2}}$$

$$\lim_{x\to\infty} -2x \to -\infty .$$

6. Let $g(x) = \int_{-2}^{x} \sin(t^5 - t)\,dt$. Find $g'(x)$. [2]

$$g'(x) = \sin(x^5 - x)$$

7. Circle to indicate whether each statement is true or false, and **justify your answers**.

(a) The most general antiderivative of $f(x) = e^{3x} - \sin x$ is $F(x) = \frac{1}{3}e^{3x} - \cos x + C$ [3]

True **False**

$$\int (e^{3x} - \sin x)\,dx = \frac{e^{3x}}{3} - (-\cos x) + c$$

$$\frac{1}{3}e^{3x} + \cos x + c$$

(b) If f is differentiable and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$. [3]

True **False**

Since f is differentiable $\Rightarrow$ implies continuous

$f(-1) = f(1)$ satisfies Rolle's theorem hypotheses

$f'(c) = 0$ $\therefore$ True.