University of Connecticut
Department of Mathematics

MATH 1131Q PRACTICE EXAM 2 SPRING 2017

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DISCUSSION SECTION: ______________________

Read This First!

- Read the questions and instructions carefully.
- The available points for each problem are given in brackets.
- You must **show your work** to obtain full credit (and to possibly receive partial credit). Correct answers with no justification will not receive credit.
- Make sure your answers are clearly indicated, and cross out any work you do not want graded.
- Do not leave any blanks! Even if you do not arrive at an answer, show as much progress towards a solution as you can, and explain your reasoning.
- Calculators are allowed, but models that can do symbolic computations (TI-89 and above, including TI-NSpire) are not allowed.

Grading - For Administrative Use Only

Page:	1	2	3	4	Total
Points:	13	9	15	13	50
Score:					

1. Find the derivative of $f'(x)$ of each of the following functions. **You do not need to simplify your answer.**

   (a) $f(x) = \dfrac{\sin x}{3e^x + 1}$ [4]

$$f'(x) = \frac{(3e^x+1)\cdot\cos x - \sin x\,(3e^x)}{(3e^x+1)^2}$$

   (b) $f(x) = 5x^8 \tan x$ [4]

$$f'(x) = 5\cdot 8x^7 \tan x + 5\cdot x^8 \cdot \sec^2 x$$

   (c) $f(x) = \cos(\ln(x^2))$ [5]

$$f'(x) = -\sin(\ln(x^2)) \cdot \frac{d}{dx}\ln(x^2)$$

$$= -\sin(\ln(x^2)) \cdot \frac{1}{x^2} \cdot 2x$$

$$= -\sin(\ln(x^2)) \cdot \frac{2}{x}$$

2. There are 30 guinea pigs introduced into an ecosystem, and their population grows at a rate proportional to its size. In 2 years, the population grows to 85 guinea pigs.

(a) Find a formula for the function $P(t)$ that gives the population of guinea pigs at any time t measured in years. [5]

$$\frac{dP}{dt} = k\cdot P$$

$P_0 = 30$ at $t=0$; $P = 85$ at $t = 2$

$$P(t) = P_0 \cdot e^{kt}$$

$$P(2) = 85 = 30 \cdot e^{k\cdot 2}$$

$$\frac{85}{30} = e^{2k}$$

$$\ln\left(\frac{85}{30}\right) = 2k$$

$$k = \frac{1}{2}\ln\left(\frac{85}{30}\right)$$

$$P(t) = 30 \cdot e^{\frac{1}{2}\ln\left(\frac{85}{30}\right)\cdot t}$$

(b) Using your answer to part (a), determine how long it would take the guinea pig population to quadruple. (means 4 times) [4]

$t = ?$ when $P = 4P_0$

$$4\cancel{P_0} = \cancel{P_0}\cdot e^{\frac{1}{2}\ln\left(\frac{85}{30}\right)\cdot t}$$

$$\ln(4) = \frac{1}{2}\ln\left(\frac{85}{30}\right)\cdot t$$

$t = 2.66$ years.

3. Let $f(x) = x^x$.

(a) Find the derivative $f'(x)$. [6]

$$y = x^x$$

$$\ln y = x \cdot \ln x$$

$$\frac{1}{y} \cdot y' = 1 \cdot \ln x + x \cdot \frac{1}{x}$$

$$y' = (\ln x + 1) y$$

$$y' = (\ln x + 1) \cdot x^x$$

(b) Use your answer to part (a) to find the linearization of $f(x)$ at $x = 1$. [3]

$$L(x) = f(a) + f'(a)(x - a) \qquad a = 1. \qquad f(a) = 1^1 = 1$$

$$f'(a) = (\overset{0}{\cancel{\ln(1)}} + 1) \cdot 1 = 1$$

$$L(x=1) = 1 + 1(x - 1)$$

$$= 1 + x - 1$$

$$L(x) = x$$

4. Find $\frac{dy}{dx}$ given $7xe^{2y} + y = 10$. **You do not need to simplify your answer.** [6]

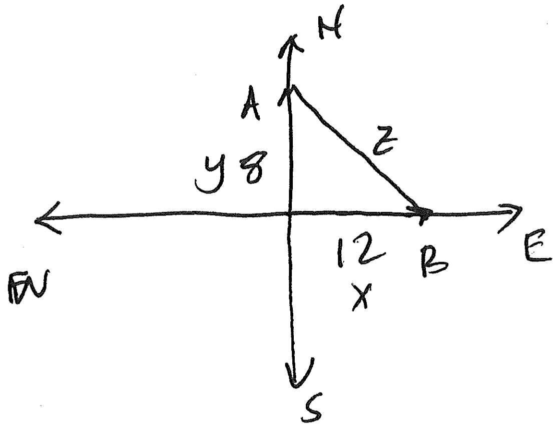
$$7 \cdot \left[1 \cdot e^{2y} + x \cdot e^{2y} \cdot 2y' \right] + y' = 0$$

$$7e^{2y} + 14xe^{2y} y' + y' = 0$$

$$(14xe^{2y} + 1) y' = -7e^{2y}$$

$$y' = \frac{-7e^{2y}}{(14xe^{2y} + 1)}$$

5. Ship A is traveling directly north away from a dock at 6 mi/hr. Ship B is traveling directly east away from the same dock at 9 mi/hr. Find the rate at which the distance between the two ships is changing at the moment when Ship A is 8 mi from the dock, and Ship B is 12 mi from the dock. [7]



$x^2+y^2=z^2$

$(18)^2+(8)^2=z^2 \quad z=\sqrt{208}.$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

Given

$\frac{dy}{dt}=6 \quad y=8$

$x=12$

$\frac{dx}{dt}=9 \quad z=\sqrt{208}$

$2\cdot(12)\cdot 9+2\cdot(8)\cdot 6=2\cdot\sqrt{208}\cdot\frac{dz}{dt}$

$216+96=2\sqrt{208}\cdot\frac{dz}{dt}$

$\frac{156}{\sqrt{208}}=\frac{dz}{dt}$ m/hr.

6. Circle to indicate whether each statement is true or false, and **justify your answers**.

(a) If $f(x)$ is a differentiable function, then $\frac{d}{dx}(f(\sqrt{x}))$ is $\frac{f'(x)}{2\sqrt{x}}$ [3]

True **False**

$\frac{d}{dx}f(\sqrt{x})=f'(\sqrt{x})\cdot\frac{d}{dx}\sqrt{x}=f'(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$

(b) If a circle is expanding and its radius is increasing at a constant rate of 2 cm/s, then its area is increasing at a constant rate. Note: the area of a circle of radius r is $A=\pi r^2$. [3]

True **False**

$A=\pi r^2$

$\frac{dA}{dt}=2\pi\cdot r\frac{dr}{dt}$

If dr/dt is constant, $\frac{dA}{dt}$ still depends on the value of 'r' @ any given instant