SOLUTIONS

University of Connecticut
Department of Mathematics

MATH 1131Q　　EXAM 1 PRACTICE　　Spring 2017

NAME: ______________________

DISCUSSION SECTION: ______________________

Read This First!

- Read the questions and instructions carefully.
- The available points for each problem are given in brackets.
- You must **show your work** to obtain full credit (and to possibly receive partial credit). Correct answers with no justification will not receive credit.
- Make sure your answers are clearly indicated, and cross out any work you do not want graded.
- Do not leave any blanks! Even if you do not arrive at an answer, show as much progress towards a solution as you can, and explain your reasoning.
- Calculators are allowed, but models that can do symbolic computations (TI-89 and above, including TI-NSpire) are not allowed.

Grading - For Administrative Use Only

Page:	1	2	3	4	Total
Points:	12	16	12	10	50
Score:					

1. Evaluate each of the following limits **using algebraic methods (no credit will be given for any other method).**

(a) $\lim_{x \to -5} \frac{\sqrt{x+14}-3}{x+5}$ [6]

$$= \lim_{x\to -5} \frac{\sqrt{x+14}-3}{x+5} \cdot \frac{\sqrt{x+14}+3}{\sqrt{x+14}+3}$$

② for multiplying by conjugate in numerator + denominator

$$= \lim_{x\to -5} \frac{x+14-9}{(x+5)(\sqrt{x+14}+3)}$$

① for computing numerator

$$= \lim_{x\to -5} \frac{x+5}{(x+5)(\sqrt{x+14}+3)}$$

$$= \lim_{x\to -5} \frac{1}{\sqrt{x+14}+3}$$

① for simplifying

$$= \frac{1}{\sqrt{-5+14}+3}$$

$$= \underline{\underline{\frac{1}{6}}}$$

② give both points if they have just the correct last line (with previous work).

work doesn't need to match exactly

(b) $\lim_{x \to \infty} \frac{2x+1-3x^2}{\sqrt{7x^4-x^2}}$ [6]

$$= \lim_{x\to\infty} \frac{\frac{2x}{x^2} + \frac{1}{x^2} - \frac{3x^2}{x^2}}{\frac{\sqrt{7x^4-x^2}}{x^2}}$$

① dividing by a power of x

① picking x^2

$$= \lim_{x\to 0} \frac{\frac{2}{x} + \frac{1}{x^2} - 3}{\frac{\sqrt{7x^4-x^2}}{\sqrt{x^4}}}$$

① simplifying numerator

$$= \lim_{x\to 0} \frac{\frac{2}{x} + \frac{1}{x^2} - 3}{\sqrt{7-\frac{1}{x^2}}}$$

① simplifying denominator

$$= \frac{0+0-3}{\sqrt{7-0}}$$

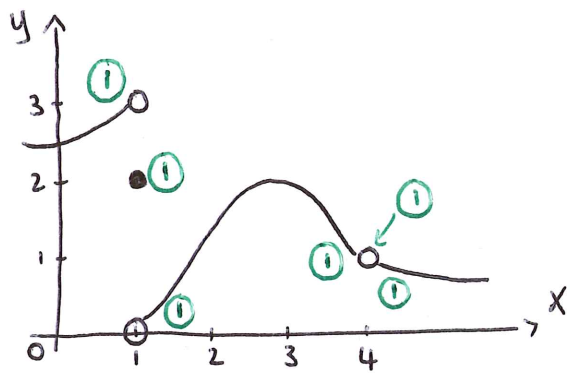
$$= \underline{\underline{-\frac{3}{\sqrt{7}}}}$$

② give both points if they have just correct last line (with previous work).

2. Let $f(x)$ be a function that satisfies all of the following:

- $\lim_{x\to 1-} f(x) = 3$, $\lim_{x\to 1+} f(x) = 0$, and $f(1) = 2$.
- $\lim_{x\to 4-} f(x) = 1$, $\lim_{x\to 4+} f(x) = 1$, and $f(x)$ is discontinuous at $x = 4$.

(a) Sketch a possible graph for $f(x)$. **Mark and label appropriate x and/or y coordinates on the axes for full credit.** [6]



-1 if $x=1$, $y=2,3$ not labeled

-1 if $x=4$, $y=1$ not labeled.

(b) Find $\lim_{x\to 1} f(x)$, or say it does not exist. Briefly justify your answer. [2]

$\lim_{x\to 1} f(x)$ DNE (1) since $\lim_{x\to 1-} f(x) = 3 \neq 0 = \lim_{x\to 1+} f(x)$ (1)

(accept anything about one-sided limits disagreeing)

(c) Is $f(x)$ continuous from the left or right or neither at $x = 1$? Briefly justify your answer. [2]

Neither (1) : $f(1) \neq \lim_{x\to 1-} f(x)$ and $f(1) \neq \lim_{x\to 1+} f(x)$ (1)

(accept anything about function value disagreeing with both one-sided limits)

3. Find all vertical asymptotes, or show that none exist, for $f(x) = \dfrac{x^2 - 6x - 7}{x^2 - 9x + 14}$. [6]

$$f(x) = \frac{(x+1)(x-7)}{(x-2)(x-7)}$$ (1) (1) > Factoring

work doesn't need to match exactly

$$\lim_{x\to 7} f(x) = \lim_{x\to 7} \frac{x+1}{x-2} = \frac{8}{5}$$

(1) for not listing $x=7$

(1) for supporting work – just simplifying to $\frac{x+1}{x-2}$ is fine.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x+1}{x-2} = \frac{3}{0} \text{ infinite}$$

vertical asymptote at $x = 2$ only

(1) for $x = 2$

(1) for supporting work – need to say something about 0 denominator.

4. Let $f(x) = \frac{8}{x^2}$.

(a) **Find $f'(x)$ using the limit definition of the derivative (no credit will be given for any other method).** [6]

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{8}{(x+h)^2} - \frac{8}{x^2}}{h}$$

② give both points if they have just 2nd line and it's correct.

$$= \lim_{h \to 0} \frac{\frac{8x^2 - 8(x+h)^2}{x^2(x+h)^2}}{h}$$

① common denominator

$$= \lim_{h \to 0} \frac{8x^2 - 8(x^2 + 2xh + h^2)}{h x^2 (x+h)^2}$$

$$= \lim_{h \to 0} \frac{\cancel{8x^2} - \cancel{8x^2} - 16xh - 8h^2}{h x^2 (x+h)^2}$$

$$= \lim_{h \to 0} \frac{h(-16x - 8h)}{h x^2 (x+h)^2}$$

$$= \lim_{h \to 0} \frac{-16x - 8h}{x^2 (x+h)^2}$$

simplifying

① $8x^2 - 8x^2$

① $\frac{h}{h}$

they don't need every detail shown here.

$$= \frac{-16x}{x^2(x+0)^2} = \frac{-16x}{x^4} = -\frac{16}{x^3}$$

①

(b) Use your answer from part (a) to find $f'(2)$. [2]

$$f'(2) = -\frac{16}{2^3} = -2$$ ① ①

(c) Use your answer from part (b) to find the equation of the tangent line to $y = \frac{8}{x^2}$ at $x = 2$. [4]

$y = \frac{8}{2^2} = 2$ ① ① so $(2,2)$ is a point on line (x_1, y_1)

slope $= m = -2$

$$y - y_1 = m(x - x_1) \Rightarrow y - 2 = -2(x - 2)$$ ②

don't have to use pt-slope form.

5. Circle to indicate whether each statement is true or false, and **justify your answers.**

(a) If $\lim_{x\to 1} g(x) = 0$ and $\lim_{x\to 1} h(x) = 0$, then $\lim_{x\to 1} \frac{g(x)}{h(x)}$ does not exist. ① giving example [3]

True **False** ① Example: If $g(x) = x-1$ and $h(x) = x-1$, ① example is correct

$\lim_{x\to 1} g(x) = \lim_{x\to 1} h(x) = 0$ but $\lim_{x\to 1} \frac{x-1}{x-1} = \lim_{x\to 1} 1 = 1 \neq 0$

$\boxed{\text{OR}}$

$\frac{0}{0}$ is indeterminate form, ① $\lim_{x\to 1} \frac{g(x)}{h(x)}$ might or might ① not exist.

(b) Let $f(x)$ be a function. If $f'(2)$ exists, then $\lim_{x\to 2} f(x) = f(2)$. [3]

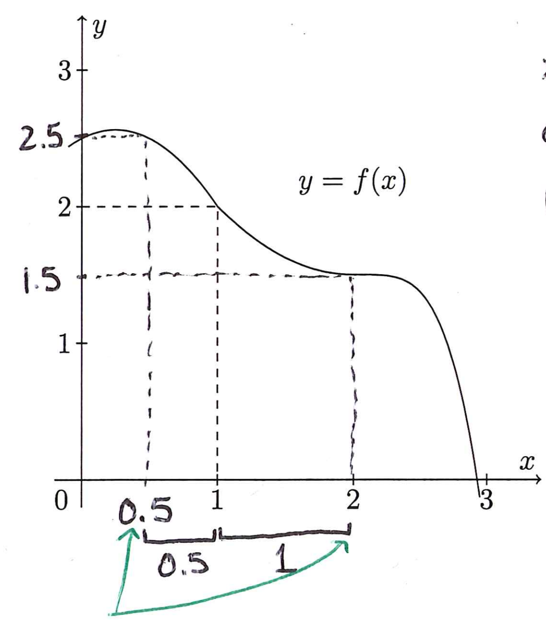
① **True** **False**

Since $f(x)$ is differentiable at $x=2$,

it is continuous ① at $x=2$.

this means ① $\lim_{x\to 2} f(x) = f(2)$

6. The graph of a function $f(x)$ is shown below. Use the graph to estimate the greatest value of δ so that if $0 < |x-1| < \delta$, then $|f(x)-2| < 0.5$. **Mark the graph to show how you calculate any values you use. Justify your answer.** [4]



x Can go up to 0.5 to left of 1 and up to 1 to right of 1 and we still have $|f(x) - 2| < 0.5$.

The furthest we can go in both directions from $x=1$ is $\delta = 0.5$ ①

① for reasonable justification.

estimating these values, marking graph ②