University of Connecticut
Department of Mathematics

MATH 1070 SAMPLE EXAM 2

Exam 2 will cover sections 6.1, 6.2, 6.3, 6.4, F.1, F.2, F.3, F.4, 1.1, and 1.2. This sample exam is intended to be used as one of several resources to help you prepare. The coverage of topics is not exhaustive, and you should look through all examples from lectures (or videos), quizzes, and homework as these will all be relevant. The wealth of problems in our text is also a good resource for practice with this material.

- The exam is a closed notes, closed book exam. You can not receive aid on this exam from anyone. Approved calculators are allowed, but there is no sharing of calculators!
- Some partial credit may be given depending on the correctness of the work submitted. You must show all work and calculations needed to reach your answers. Just using a calculator is not sufficient for credit.
- Please make sure to attend the exam that you signed up for at the beginning of the term. The room that your exam is in can be found on the common course webpage.
- Please note that the table on the last page of the sample exam will also be included on the exam.

1. A baseball player has a batting average of 0.47 (this is the probability of getting a hit each time they bat). The player bats 4 times in a game. Let X be the number of hits the player gets in a game.

   (a) What is the probability that the player gets exactly 2 hits?

   Bernoulli with $n=4$, $p=.47$, $q=1-p=.53$

   $P(X=2) = C(4,2)(.47)^2(.53)^2 \approx .3723$

   (b) Draw and fill in a probability distribution table for the random variable X.

x	0	1	2	3	4
$P(X=x)$	.0789	.2799	.3723	.2201	.0488

   Note: $P(X=0) \neq P(X=4)$, $P(X=1) \neq P(X=3)$ (unlike coin tossing)

   (c) What is the player's expected number of hits? Explain your answer.

   $E(X) \approx 0(.0789) + 1(.2799) + 2(.3723) + 3(.2201) + 4(.0488)$

   ≈ 1.88

   (Note, for a Bernoulli random variable X, we actually get $E(X) = \mu = np = 4(.47) = 1.88$)

(This is not something you need to know!)

1 2 3 4 5 6 7 8 9 10

2. Let $S = \{94, 97, 82, 68, 74, 83, 85, 91, 77, 69\}$. Find the mean, variance, and standard deviation of the values in S.

$$\mu = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{94+97+82+68+74+83+85+91+77+69}{10}$$

$$= 82$$

$$\sigma^2 = \frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \ldots + (x_n-\mu)^2}{n}$$

$$= \frac{(94-82)^2 + (97-82)^2 + (82-82)^2 + (68-82)^2 + (74-82)^2 + (83-82)^2 +}{}$$

$$\ldots \frac{(85-82)^2 + (91-82)^2 + (77-82)^2 + (69-82)^2}{10}$$

$$= 91.4$$

$$\therefore \sigma = \sqrt{91.4} \approx 9.56$$

3. A machine produces screws with diameters which are normally distributed. The mean diameter of a screw is 0.224 inches and the standard deviation is 0.004 inches. Quality requirements demand a screw to be rejected if the diameter is more than 0.001 inches different from the mean.

$\mu = .224, \quad \sigma = .004$

(a) Find $P(X \leq 0.221)$, where X is the diameter of the screw.

$$P(X \leq .221) = P\left(Z \leq \frac{.221 - .224}{.004}\right) = P(Z \leq -.75)$$

$$\approx .2266$$

Terrible quality control!

(b) What is the probability of a screw being rejected?

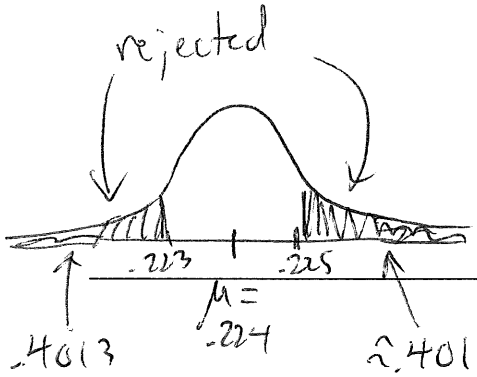
$$P(\text{rejected}) = 1 - P(\text{not rejected})$$

$$\approx 1 - .1974 = .8026$$

$$P(\text{not rejected}) = P(.223 \leq X \leq .225) = P(-.25 \leq Z \leq .25)$$

$$= P(Z \leq .25) - P(Z \leq -.25) \approx .1974$$

$$.5987 \qquad .4013$$



-or-

same

$$P(\text{rejected}) = P(Z \leq -.25) + P(Z \geq .25)$$

$$= 2 \cdot P(Z \leq -.25)$$

$$\approx 2 \cdot .4013 = .8026$$

4. Eighteen years ago, your aunt gave you an 18-year bond with an annual interest rate of 7.5% compounded quarterly. The bond is currently worth \$10,000. What was the price of the bond when your aunt purchased it?

Compound Interest $\Rightarrow F = P\left(1+\frac{r}{m}\right)^{mt}$, $P = ?$

$\Rightarrow F = 10{,}000 = P\left(1+\frac{.075}{4}\right)^{4\cdot 18}$

$\hookrightarrow P \approx \frac{10{,}000}{3.809507} \approx \$2{,}625.01$

5. An individual is seeking to purchase a house in 10 years time. She wishes to save \$150,000 for a down payment on a future house. She has found an account which offers an annual interest rate of 4.5% compounded monthly.

(a) How much should she deposit into this account each month to reach her goal of \$150,000 in 10 years?

Ordinary annuity & future value $\Rightarrow FV = PMT \times \frac{(1+i)^n - 1}{i}$, $PMT = ?$

$FV = 150{,}000 = PMT \times \frac{\left(1+\frac{.045}{12}\right)^{12\cdot 10} - 1}{\left(\frac{.045}{12}\right)}$

$\hookrightarrow PMT \approx \$992.08$

(b) How much total interest does this annuity earn in 10 years?

Total paid $= \$992.08 \times 12 \times 10 = \$119{,}049.60$

Interest $= \$150{,}000 - \$119{,}049.60 = \$30{,}950.40$

6. A lottery has one \$100,000 prize, two \$25,000 prizes, three \$5,000 prizes, and ten \$500 prizes. There are 100,000 lottery tickets sold at \$2 each, and each is equally likely to win. Find the expected return on buying one lottery ticket.

Let X = amount won

x	100,000	25,000	5,000	500	0
$P(X=x)$	$\frac{1}{100,000}$	$\frac{2}{100,000}$	$\frac{3}{100,000}$	$\frac{10}{100,000}$	$\frac{99,984}{100,000}$

$$E(X) = 100,000\left(\frac{1}{100,000}\right) + 25,000\left(\frac{2}{100,000}\right) + 5,000\left(\frac{3}{100,000}\right) + 500\left(\frac{10}{100,000}\right) + 0(\quad)$$

$$= 1.7$$

$\therefore$ Expected return = (expected winnings) − (cost to play)

$$= 1.7 - 2$$

$$= -\$.3$$

(that is, lose 30 cents on average each time)

7. A family obtains a loan of \$30,000 to buy a new car. Suppose that the annual interest rate on the loan is 8.7%, and that the family will pay off the loan over the course of 5 years.

(a) What will the family's monthly payment be?

Ordinary annuity & present value $\Rightarrow PV = PMT \times \frac{1-(1+i)^{-n}}{i}$, $PMT = ?$

$$PV = 30{,}000 = PMT \times \frac{1-\left(1+\frac{.087}{12}\right)^{-12\cdot 5}}{\left(\frac{.087}{12}\right)}$$

$$\Rightarrow PMT \simeq \$618.39$$

(b) What is the total interest the family will pay on the loan?

Total paid $= \$618.39 \times 12 \times 5 = \$37{,}103.40$

Interest $= \$37{,}103.40 - \$30{,}000$

$= \$7{,}103.40$

8. A diner finds that there is demand for 96 burgers when they are priced at \$5.15 each, while the demand increases to 120 when they are priced at \$3.95 each. Assume linear supply and demand models.

(a) Find the demand equation. Write your answer in the form $p = mx + b$.

$(96, 5.15), (120, 3.95)$

$m = \frac{3.95 - 5.15}{120 - 96} = \frac{-1.2}{24} = -.05$

Use either point to find b. Use $(96, 5.15)$:

$5.15 = -.05(96) + b \Rightarrow b = 5.15 + .05(96) = 9.95$

$\therefore$ Demand equation is $p = -.05x + 9.95$

(b) If the supply equation is $p = 0.1x + 2.45$, find the equilibrium price and quantity.

Equilibrium $\Rightarrow$ supply = demand

$-.05x + 9.95 = .1x + 2.45$

$\Rightarrow 7.5 = .15x$

$\Rightarrow x = 50 =$ equilibrium quantity

$p = -.05(50) + 9.95$

$= 7.45 =$ equilibrium price

(Check $p = .1(50) + 2.45 = 7.45$ ✓)

9. Suppose a firm manufacturing microwaves has a fixed cost of \$4,000 and an additional cost of \$75 for each microwave produced. Assume linear cost and revenue models.

(a) Find the cost equation.

Let x = number of microwaves

$$C(x) = C = 4{,}000 + 75x$$

fixed costs: 4,000 — variable costs: 75x

(b) If the profit from selling 100 microwaves is \$7,250 find the revenue equation.

$$C(100) = 4{,}000 + 75(100) = 11{,}500$$

$$P = R - C \Rightarrow R(100) = P(100) + C(100)$$

$$= 7{,}250 + 11{,}500$$

$$= 18{,}750$$

Since $R(x) = mx$, $R(100) = 18{,}750 \Rightarrow m = 187.5$

$$\therefore R(x) = R = 187.5x$$

Helpful formulas:

Simple Interest

$$F = P(1 + rt)$$

Compound Interest

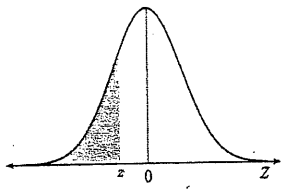
$$F = P(1+i)^n = P\left(1 + \frac{r}{m}\right)^{mt} \qquad r_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1$$

Ordinary Annuities

$$FV = PMT \times \frac{(1+i)^n - 1}{i} \qquad PV = PMT \times \frac{1 - (1+i)^{-n}}{i}$$

Area Under a Normal Curve

This table gives the area under the standard normal curve to the left of $z = \frac{x-\mu}{\sigma}$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9984	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998