Math 1070 Exam #1 Review

1. You are being dealt five cards from a standard deck of 52 playing cards. Find the probability that you are dealt two clubs and three red cards.

Clubs and reds are mutually exclusive, so;

$$P(2 \text{ Clubs and } 3 \text{ red}) = \frac{{}_{13}C_2 \cdot {}_{26}C_3}{{}_{52}C_5}$$

$$= \frac{202{,}800}{2{,}598{,}960} \approx 0.078$$

2. A dartboard has a radius of 9 in. and is composed of a bull's eye with a radius of 1 in. surrounded by four concentric rings, each has a width of 2 in. If you randomly throw a dart at the board (and hit it), find the probability of hitting each individual ring as well as the bull's eye. (*Hint: Picture?*)

$$P(\text{Bull's eye}) = \frac{\pi(1)^2}{\pi(9)^2} = \frac{1}{81}$$

Going outward:

$$P(1^{st} \text{ ring}) = \frac{\pi(3)^2}{\pi(9)^2} - \frac{1}{81} = \frac{8}{81}$$

$$P(2^{nd} \text{ ring}) = \frac{\pi(5)^2 - \pi(3)^2}{\pi(9)^2} = \frac{16}{81}$$

$$P(3^{rd} \text{ ring}) = \frac{\pi(7)^2 - \pi(5)^2}{\pi(9)^2} = \frac{24}{81}$$

$$P(4^{th} \text{ ring}) = \frac{\pi(9)^2 - \pi(7)^2}{\pi(9)^2} = \frac{32}{81}$$

$$\Sigma = 1 \checkmark$$

3. A plane has crashed in one of three equally likely, different regions. Region 1 is a wooded area, region 2 is a relatively flat farming area, and region 3 is a hilly area. Searchers choose to start looking in region 2 because is it the easiest to search and they believe that if a plane has crashed in this region, the probability that they find it is 0.9. If they search region 2 and do not locate the plane, find the conditional probability that the plane is in region 3. (*Hint: You need a compliment*)

Using Bayes' Formula: Want $P(\text{crash in } R3 \mid \underbrace{\text{search and don't find in } R2}_{\text{event } F})$

$R_1 \Rightarrow$ Crash in Region 1

$R_2 \Rightarrow$ " " 2

$R_3 \Rightarrow$ " " 3

$P(F) = 1 - 0.9 = 0.1$

$$\text{So, } P(R3 \mid F) = \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot (0.1) + \frac{1}{3} \cdot 1}$$

$$= \frac{10}{21} \approx 0.4762$$

4. Suppose that there are three corporations competing for four different government contracts. If the contracts are awarded randomly, what is the probability that each corporation will get a contract?

Sample Space $\Rightarrow$ 3^4 $\div$ 3 choice for all four contracts

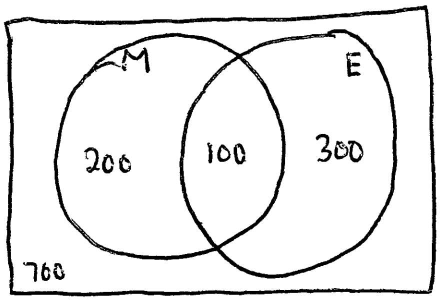
$$\text{Event space} \Rightarrow \binom{3}{1}\binom{4}{2}\binom{2}{1}\binom{1}{1} = 36$$

Who gets 2 — which 2? — 2 contracts choices for 1 comp — 1 contract choice for 1 comp

$$P(\text{each get at least } 1) = \frac{36}{3^4} = 0.\overline{44}$$

5. The registrar reported that among 1300 students, 700 students did not register for either a math or English course, 400 registered for an English course, and 300 registered for both types of courses.

a) How many registered for an English course but not a Math course?



300 Registered for ~~math~~ English

b) How many registered for a math course?

200 Registered for Math

6. Find the indicated sets with:

$U = \{1,2,3,4,5,6,7,8,9,10\}, A = \{1,2,3,4,5,6,\}, B = \{4,5,6,7,8\}, C = \{5,6,7,8,9,10\}$

Double check theses!

$A^c = \{7,8,9,10\}$ $B^c = \{1,2,3,9,10\}$ $C^c = \{1,2,3,4\}$

a) $A \cap B \cap C$

$\{5,6\}$

b) $A \cap (B^c \cup C)$

$\{1,2,3,5,6\}$

c) $(A \cup B \cup C)^c$

$\emptyset$

d) $A^c \cap B^c \cap C$

$\{9,10\}$

e) $A \cup (B \cap C)$

$\{1,2,3,4,5,6,7,8\}$

7. Callahan Auto Parts has hired 12 new employees, and must assign 8 to the day shift and 4 to the night shift. Assume that the 12 employees consist of 6 men and 6 women and that the assignments to day and night shift are made at random.

(a) What is the probability that all 4 of the night-shift employees are men?

$$P(\text{night all men}) = \frac{6C4}{12C4} = \frac{15}{495} = \frac{1}{33} = 0.\overline{03}$$

(b) What is the probability that at least one of the night-shift employees is a woman?

$$\begin{aligned} P(\text{night at least one women}) &= 1 - P(\text{no women}) \\ &= 1 - P(\text{night all men}) \\ &= 1 - \tfrac{1}{33} = 0.\overline{96} \end{aligned}$$

8. A small island has three bridges connecting it to the main land. In the next year, bridge A has a 15% chance of collapsing, bridge B has a 5% chance of collapsing, and bridge C has a 20% chance of collapsing. What is the probability that exactly one bridge will collapse next year?

$$\left.\begin{aligned} P(\text{Bridge A collapse only}) &= (0.15)(0.95)(0.8) = 0.114 \\ P(\text{'' B ''}) &= (0.85)(0.05)(0.8) = 0.034 \\ P(\text{'' C ''}) &= (0.85)(0.95)(0.2) = 0.1615 \end{aligned}\right\} \text{SUM?}$$

0.3095

$$\cancel{P(\text{exactly } 1) = \frac{0.114 + 0.034 + 0.1615}{(0.0015 + 0.006 + 0.0285 +}}$$