University of Connecticut
Department of Mathematics

MATH 1071 EXAM 1 FALL 2016

NAME: Solutions

Instructor Name: _____ Section: _____

Read This First !

- Please read each question carefully. Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- Calculators are allowed, however models TI-89 and above are not permitted.

Grading - For Administrative Use Only

Page:	1	2	3	4	5	6	7	Total
Points:	16	16	14	12	16	14	12	100
Score:								

1. Solve the following inequality, and write your answer in interval notation. [6]

$$0 \leq 1 - 3x < 10$$

$0 \leq 1-3x < 10$ 1 pt
$-1 \quad -1 \quad -1$

$-1 \leq -3x < 9$ 2 pts
$\frac{}{-3} \quad \frac{}{-3} \quad \frac{}{-3}$

$\frac{1}{3} \geq x > -3$ 2 pts

$(-3, \frac{1}{3}]$ 1 pt

2. Simplify the following to a form that does not contain radicals or negative exponents.

(a) [6]

$$\sqrt[7]{\frac{x^3 y^{14}}{z^{-70} x^{-4}}}$$

$(x^3 y^{14} z^{70} x^4)^{1/7}$ 2 pts

$(x^7 y^{14} z^{70})^{1/7}$ 2 pts

$x y^2 z^{10}$ 2 pts

$xy^2 z^{10}$

(b) [4]

$$8^{2 \log_8(3)}$$

$8^{\log_8(3^2)} = 3^2$

2 pts 2 pts

9

3. Find the domain of the following functions: Interval Notation?

(a) $f(x) = \dfrac{x^2+1}{\sqrt{9-x}}$ [4]

$9-x > 0$ 2pts

$-9 \quad -9$

$-x > -9$ 1pt

$x < 9$ 1pt

$(-\infty, 9)$

(b) $f(x) = \dfrac{x+4}{x^2-2x-24} = \dfrac{x+4}{(x-6)(x+4)}$ [4]

$x-6=0 \quad x+4=0$ 2pts

$x=6 \quad x=-4$ 1pt

$(-\infty,-4) \cup (-4,6) \cup (6,\infty)$ 1pt

4. Evaluate the expression $\dfrac{f(a+h)-f(a)}{h}$ given that $f(x) = \dfrac{1}{2x+5}$ and simplify as much as possible. [8]

$$\frac{1}{h}\left(\frac{1}{2(a+h)+5} - \frac{1}{2a+5}\right) = \frac{1}{h}\left(\frac{2a+5-(2(a+h)+5)}{(2a+5)(2(a+h)+5)}\right)$$

2pts 2pts

$$= \frac{2a+5-2a-2h-5}{h(2a+5)(2(a+h)+5)} = \frac{-2h}{h(2a+5)(2(a+h)+5)}$$

2pts

$$= \boxed{\frac{-2}{(2a+5)(2(a+h)+5)}}$$

2pts

5. The revenue and cost functions for a product are given below. The revenue and cost are given in dollars and x represents the number of units.

$$\text{Revenue: } R(x) = -2x^2 + 15x$$

$$\text{Cost: } C(x) = 5x + 8$$

(a) What is the profit function, $P(x)$? [3]

$P = R - C$

$P = -2x^2 + 15x - (5x + 8)$ 1 pt

$P = -2x^2 + 15x - 5x - 8$ 1 pt

$\boxed{P = -2x^2 + 10x - 8}$ 1 pt

(b) At what production level(s) will the company break even? [6]

$P = -2(x^2 - 5x + 4)$ or $x = \dfrac{-10 \pm \sqrt{100 - 4(-2)(-8)}}{2(-2)}$

$0 = -2(x-4)(x-1)$ 2 pts

$= \dfrac{-10 \pm \sqrt{100 - 64}}{-4}$

$x - 4 = 0 \quad x - 1 = 0$ 2 pts

$= \dfrac{-10 \pm \sqrt{36}}{-4}$

$\boxed{x = 4 \quad x = 1}$ 2 pts

$= \dfrac{-10 \pm 6}{-4}$

$= \dfrac{-16}{-4}, \dfrac{-4}{-4}$

$\boxed{x = 4, 1}$

(c) Find the value of x that maximizes the profit. [5]

$x = \dfrac{-b}{2a} = \dfrac{-10}{2(-2)} = \dfrac{10}{4} = \dfrac{5}{2}$

2 pts 1 pt 2 pts

$\boxed{x = 5/2 = 2.5}$

6. (a) The manufacturers are willing to supply 100 of these products when the price is set at \$30. If the price is increased to \$45, the manufacturers have agreed to supply 60 more. Find the linear supply equation for this product [6]

x P
(100, 30)
(160, 45)
2pts

$m = \frac{45-30}{160-100} = \frac{15}{60} = \frac{1}{4}$ 2pts

$P - 30 = \frac{1}{4}(x - 100)$ 2pts

$P = \frac{1}{4}x - 25 + 30$

$\boxed{P = \frac{1}{4}x + 5}$

(b) The demand equation for a particular product is known to be $p = -x + 25$. Find the equilibrium point using the demand equation and the supply equation you found in part (a). [6]

$\frac{1}{4}x + 5 = -x + 25$ 1pts

$\frac{5}{4}x = 20$ 1pts

$x = 20\left(\frac{4}{5}\right)$

$\boxed{x = 16}$ 2pts

$P = \frac{1}{4} \cdot 16 + 5$ 1 pt

$\boxed{P = 9}$ 1pt

7. In the following problems, solve for x. Give an exact answer.

(a) $4^{2x} = 8^{9x+15}$ [6]

1pt each

$(2^2)^{2x} = (2^3)^{9x+15}$ 2pts

$4x = 27x + 45$ 2pts

$-27x \quad -27x$

$-23x = 45$

$x = \frac{-45}{23}$ This is -1.9565

2pts for the answer

(b) $\ln(x) = 2\ln(3) + \ln(5) - \ln(15)$ [6]

$\ln(x) = \ln(3^2) + \ln(5) - \ln(15)$ 1pt

$\ln(x) = \ln(9 \cdot 5) - \ln(15)$ 1pt

$\ln(x) = \ln\left(\frac{45}{15}\right)$ 1pt

$\ln(x) = \ln(3)$ 1pt

$x = 3$ 2pts

(c) $5 \cdot (7^x) + 1 = 16$ [4]

$-1 \quad -1$

$\frac{5 \cdot 7^x}{5} = \frac{15}{5}$ 1pt

$7^x = 3$ 1pt

$\log_7(7^x) = \log_7(3)$ 1pt

$x = \log_7(3) = \frac{\ln(3)}{\ln(7)}$ this is 0.5646 1pt

8. Evaluate: [6]

$$\lim_{x\to 5} \frac{x^2+x-30}{x^2-7x+10}$$

$\lim_{x\to 5} \frac{(x+6)(x-5)}{(x-2)(x-5)} = \lim_{x\to 5} \frac{(x+6)}{(x-2)} = \boxed{\frac{11}{3}}$ 2pts

2pts 2pts

This is $3.\overline{6}$

9. Given the function $f(x) = -\frac{12}{x+3}$

(a) Evaluate [4]

$$\lim_{x\to -3^-} f(x)$$

$\lim_{x\to -3^-} \frac{-12}{(x+3)} = \infty$ 4pts

$\frac{-12}{(x+3)} \to \frac{-12}{0^-}$

~~DNE~~

(b) Evaluate [4]

$$\lim_{x\to -3^+} f(x)$$

$\lim_{x\to -3^+} \frac{-12}{(x+3)} = -\infty$ 4pts

$\frac{-12}{(x+3)} \to \frac{-12}{0^+}$

~~DNE~~

10. In interval notation, state the values of x for which the function $f(x)$ below is continuous. [8]

$$f(x) = \begin{cases} 3x+1 & \text{if } x < 3, \\ x^2 - 1 & \text{if } 3 \le x \le 5, \\ 24 & \text{if } x > 5 \end{cases}$$

1pt $3(3)+1 = 7$
1pt $3^2 - 1 = 8$ not continuous at $x=3$ 1pt

1pt $5^2 - 1 = 24$
1pt 24 $(x>5)$ continuous at $x=5$ 1pt

$(-\infty, 3) \cup (3, \infty)$ 2pts

11. You just received a 10-year savings bond with an annual interest rate of 9% compounded continuously. In ten years the bond will be worth \$2,000. How much is the bond worth now? [4]

$F = Pe^{rt}$

$2000 = Pe^{.09(10)}$ 2pts

$P = \frac{2000}{e^{.9}} \approx 813.139$ \$813.14

1pt 1pt