Name: _____

Score: _____ /20

Triple Integrals

Please staple your work and use this page as a cover page.

1. Compute the iterated integral.
   (a) $\displaystyle\int_0^2 \int_0^1 \int_0^3 (xy + z^2)\, dz\, dy\, dx$
   (b) $\displaystyle\int_0^2 \int_0^{z^2} \int_0^{y-z} (2x - y)\, dx\, dy\, dz$
   (c) $\displaystyle\int_0^{\pi/2} \int_0^{y} \int_0^{x} \cos(x + y + z)\, dz\, dx\, dy$
2. Compute $\displaystyle\iiint_E xy\, dV$, where $E = \{(x, y, z) \mid 0 \leqslant x \leqslant 3,\ 0 \leqslant y \leqslant x,\ 0 \leqslant z \leqslant x + y\}$.
3. Compute $\displaystyle\iiint_T x^2\, dV$, where T is the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.
4. Sketch the solid whose volume is given by the iterated integral.
   (a) $\displaystyle\int_{-1}^2 \int_0^4 \int_{-2}^3 dz\, dy\, dx$ (and find the volume)
   (b) $\displaystyle\int_0^1 \int_0^{1-x} \int_0^{2-2z} dy\, dz\, dx$
   (c) $\displaystyle\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy$
5. Write the other 5 integrals that are equivalent to
   $$\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx.$$
6. Compute $\displaystyle\iiint_E yz\, dV$, where E is the region above $z = 0$, below $z = y$, and inside $x^2 + y^2 = 4$.
7. Compute $\displaystyle\iiint_E y^2 z^2\, dV$, where E is the region bounded by $x = 1 - y^2 - z^2$ and $x = 0$.

   Hint: $\displaystyle\int \cos^2 x \sin^2 x\, dx = \frac{1}{32}(4x - \sin(4x)) + C$
8. Sketch the solid whose volume is given by the iterated integral.
   (a) $\displaystyle\int_{\pi/2}^{3\pi/2} \int_0^4 \int_{-1}^2 r\, dz\, dr\, d\theta$ (and find the volume)
   (b) $\displaystyle\int_{-\pi/2}^{\pi/2} \int_0^2 \int_0^{r^2} r\, dz\, dr\, d\theta$
   (c) $\displaystyle\int_0^2 \int_0^{2\pi} \int_0^{r} r\, dz\, d\theta\, dr$
9. Find the volume contained above $z = x^2 + y^2$ and below $z = \sqrt{x^2 + y^2}$.

10. Compute the following integral by making a change in coordinates.

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2+y^2}\, dz\, dy\, dx$$

11. Compute $\iiint_E (x^2+y^2+z^2)^{7/2}\, dV$, where $E = \{(x,y,z) \mid x \leqslant 0,\ y \leqslant 0,\ z \geqslant 0,\ 4 \leqslant x^2+y^2+z^2 \leqslant 16\}$.

12. Sketch the solid whose volume is given by the iterated integral.

    (a) $\displaystyle\int_0^{2\pi} \int_0^{\pi/2} \int_0^{3} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$ (and find the volume)

    (b) $\displaystyle\int_0^{\pi/6} \int_0^{\pi/2} \int_0^{3} \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$

    (c) $\displaystyle\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^{2} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

13. Compute the following integral by making a change in coordinates.

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} x^2 \sqrt{x^2+y^2+z^2}\, dz\, dx\, dy.$$