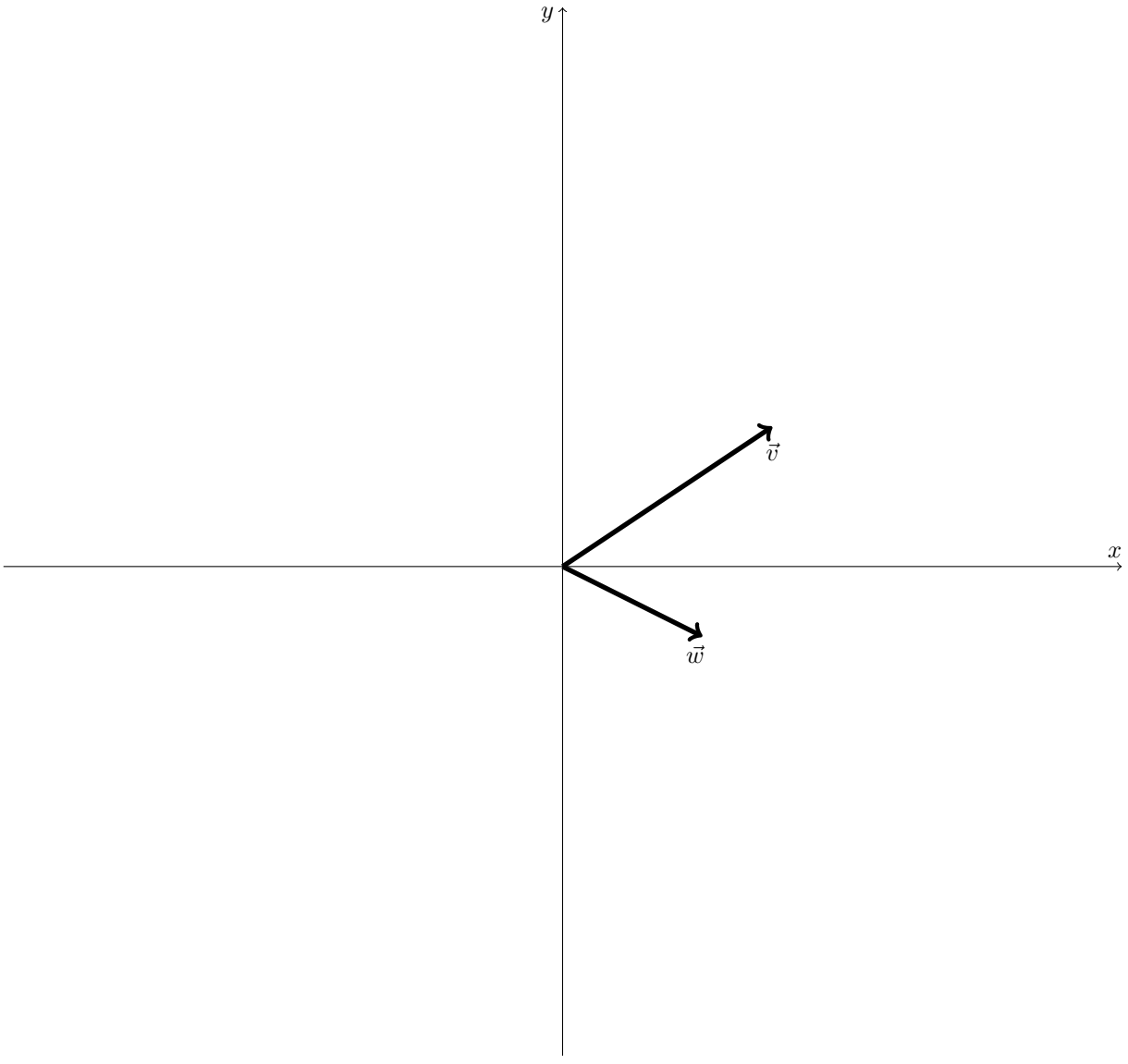
Name: _____

Score: _____ /20

Practice with Vectors

Please staple your work and use this page as a cover page.

1. On the given set of axes, sketch $\vec{v} + \vec{w}$, $\vec{w} - \vec{v}$, $2\vec{v}$, $-3\vec{w}$, and $2\vec{v} - 3\vec{w}$.



2. Two forces $\vec{F}_1 = \langle 3, 2, -7 \rangle$ and $\vec{F}_2 = \langle 12, -4, 3 \rangle$ are acting on an object. What are the net forces acting on the object in the x-, y-, and z-directions?

3. James Bond is chasing Mr. Green, who runs onto a pier and takes off in a boat. Bond, right behind him, jumps into a speedboat and starts to hotwire it. Mr. Green drives his boat along the vector $\langle 2, 5 \rangle$ and then sharply turns to the left. Bond starts the boat and zooms off, and, being the expertly-trained spy that he is, determines that Mr. Green will move along $\langle -4, 1 \rangle$. In what direction should Bond drive to intercept Mr. Green before he gets away?

4. A standard clock has an hour hand that is 2 in long. The hour hand is currently pointing at the 3, that is, the tip of the hour hand has position vector $\langle 2, 0\rangle$. Find the vector giving the displacement of the tip of the hour hand if

   (a) the hour hand moves from pointing at the 3 to pointing at the 8.

   (b) the hour hand moves from pointing at the 3 to pointing at the 1.

   (c) Find the magnitudes of the displacement vectors that you found in (a) and (b).

   (For example, 3 can be represented by $\langle 2, 0\rangle$ and 6 can be represented by $\langle 0, -2\rangle$, and the displacement of the hour hand moving from 3 to 6 is given by $\langle 0, -2\rangle - \langle 2, 0\rangle = \langle -2, -2\rangle$.)

5. Let $\vec{v} = \langle 1, 11, 3\rangle$ and $\vec{w} = \langle -2, 8, 6\rangle$. Find $\vec{v} - \vec{w}$ and the unit vector $\vec{u}$ pointing in the direction of $\vec{v} - \vec{w}$.

6. Find the angle between each given pair of vectors.

   (a) $\vec{a} = \langle 2, 7\rangle$, $\vec{b} = \langle 3, -1\rangle$

   (b) $\vec{v} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\vec{w} = -5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

7. Find a vector that is orthogonal to each given pair of vectors.

   (a) $\vec{a} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$, $\vec{b} = 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$

   (b) $\vec{v} = \langle 10, 5, -3\rangle$, $\vec{w} = \langle 4, 7, 2\rangle$

8. James Bond is in a boat located at $(1, 6)$ and perceives that Mr. Green's boat is located at $(-4, -2)$. If Bond's boat is currently facing in the direction $\langle -1, 2\rangle$, find the angle the boat must turn through to be facing in the direction of Mr. Green's boat.

9. Suppose we apply a force $\vec{F} = \langle -6, 1, 11\rangle$ (force is in Newtons) to move an object from $(4, -3, -3)$ to $(8, 1, 7)$ (distance is in m). Compute the work done by the force.

10. Consider the vector $\vec{a} = \langle -4, 5, 2\rangle$.

    (a) Find a vector $\vec{b}$ that is orthogonal to $\vec{a}$ (there are infinitely many possibilities!).

    (b) Find a vector $\vec{c}$ that is orthogonal to both $\vec{a}$ and $\vec{b}$.

11. Assume that an airplane is represented by the point $(-1, 3)$ and that a phone tower is represented by $(3, -2)$. If the plane travels in the direction of $\langle 3, -4\rangle$, will the plane collide with the tower or not? Explain your answer.

12. A particle moves in the direction $\langle 1, -3, 2\rangle$. If a force of $\langle 2a, a, 3\rangle$ Newtons is applied to the particle, for what value(s) of the constant a will the total work done by the force be zero?