Name: ____________________

Score: ________ /20

Vector Functions and Parameterized Curves

Please staple your work and use this page as a cover page.

1. Find the value(s) of t for which the curve given by $\vec{r}(t) = \langle t^2, 1-3t, 1+t^3 \rangle$ passes through the points $(1, 4, 0)$ and $(9, -8, 28)$. Also, show that the curve does not pass through the point $(4, 7, -6)$.

2. We don't know the equation that defines a certain surface S, but we are able to determine the equation of two curves that lie in the surface and that intersect at the point $(2, 1, 3)$, namely

$$\vec{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t+t^2 \rangle \qquad \text{and} \qquad \vec{r}_2(t) = \langle 1+t^2, 2t^3-1, 2t+1 \rangle.$$

   Determine an equation for the tangent plane at the point $(2, 1, 3)$.

3. Show that the curve given by $\vec{r}(t) = \langle (1+\cos 16t)\cos t, (1+\cos 16t)\sin t, 1+\cos 16t \rangle$ lies on the cone $z = \sqrt{x^2+y^2}$.

4. We say that two curves $\vec{r}_1(t)$ and $\vec{r}_2(t)$ *intersect* if they ever pass through the same point (this could be at different times) and that they *collide* if they cross at the same time.

   For example, the curves $\vec{r}_1(t) = \langle t-1, 0 \rangle$ and $\vec{r}_2(t) = \langle \cos t, \sin t \rangle$ intersect at points $(-1, 0)$ and $(1, 0)$, but they do not collide. Before continuing, think about why that is true.

   Say that two missiles are fired with trajectories given by the vector functions

$$\vec{r}_1(t) = \langle t^2, 7t-12, t^2 \rangle \qquad \text{and} \qquad \vec{r}_2(t) = \langle 4t-3, t^2, 5t-6 \rangle.$$

   Assuming $t \geqslant 0$, will the missiles collide?

   Hint: It may be helpful to rename the variable in the second trajectory as s, namely $\vec{r}_2(s) = \langle 4s-3, s^2, 5s-6 \rangle$. The paths of the missiles will intersect if you can find a pair (t, s) so that each curve passes through the same point, and the missiles will collide if that point is reached when $t = s$.

5. Two particles travel along paths given by

$$\vec{r}_1(t) = \langle t, t^2, t^3 \rangle \qquad \text{and} \qquad \vec{r}_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle.$$

   Do their paths intersect? If so, will the particles collide?

6. Find an equation for the tangent line to the curve $\vec{r}(t) = \langle e^t, te^t, te^{t^2} \rangle$ at the point $(1, 0, 0)$.

7. A bee flies along the path $\vec{r}(t) = \langle 12t, 8t^{3/2}, 3t^2 \rangle$.
   (a) Find the displacement of the bee from $t = 0$ to $t = 1$.
   (b) Find the distance traveled by the bee from $t = 0$ to $t = 1$.

8. Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the length of the portion of C from the origin to the point $(6, 18, 36)$.

   Hint: Start by finding a parameterization $\vec{r}(t)$ for the curve of intersection of these two surfaces.