Name: _____

Score: _____ /20

Understanding Surfaces

Please staple your work and use this page as a cover page.

1. For each of the following, describe all possible intersections of the given types of surfaces. If it is possible for the surfaces to not intersect, be sure to mention this and explain why.
   (a) Two lines
   (b) A line and a plane
   (c) Two planes
   (d) Three planes
   (e) A plane and a circular cylinder
   (f) A plane and a sphere
   (g) Two spheres
   (h) An elliptic paraboloid and a plane
   (i) A (true) cone and a plane

2. Say that we have a sphere centered at the origin and a half cone with its vertex on the z-axis that opens vertically and symmetrically about the z-axis (like an infinitely-tall, right, circular cone). If the vertex is moved up and down along the z-axis, how does the intersection of the cone and the sphere change? List and explain all possibilities. Also, explain any changes if the radius of the sphere is increased or decreased.

3. What is the difference between $x^2 + y^2 = 9$ and $x^2 + y^2 \leqslant 9$? What do they look like? Do they both have surface area? Volume?

4. Sketch the region given by $1 \leqslant x^2 + y^2 \leqslant 4$, $z \leqslant 0$.

5. Assume that gravity acts in the direction of the negative z-axis. If we poured water from high above the xy-plane, which of the surfaces $z = x^2$, $z = y^2$, $x = y^2$, and $y = z^2$ would hold the water? Why or why not?