University of Connecticut
Department of Mathematics

MATH 2110Q | PRACTICE EXAM 3 | FALL 2016

NAME: SOLUTIONS

DISCUSSION SECTION: ______

Read This First!

- Read the questions and instructions carefully.
- The available points for each problem are given in brackets.
- You must **show your work** to obtain full credit (and to possibly receive partial credit). Correct answers with no justification will not receive credit.
- Make sure your answers are clearly indicated, and cross out any work you do not want graded.
- Do not leave any blanks! Even if you do not arrive at an answer, show as much progress towards a solution as you can, and explain your reasoning.
- Calculators are allowed, but models that can do symbolic computations (TI-89 and above, including TI-NSpire) are not allowed.

Grading - For Administrative Use Only

Page:	1	2	3	4	5	Total
Points:	11	8	11	12	8	50
Score:						

1. A particle moves along the curve given by $\vec{r}(t) = \langle \ln(t), (t+1)^2, 3-t \rangle$, $t > 0$.

(a) Find $\vec{r}\,'(t)$. [3]

$\vec{r}\,'(t) = \langle \frac{1}{t}, 2(t+1), -1 \rangle$

(b) Find an equation of the tangent line at the point $(0, 4, 2)$. $\Rightarrow t = 1$ [4]

$\vec{r}\,'(1) = \langle 1, 4, -1 \rangle$

$\therefore \vec{L}(t) = \langle 0, 4, 2 \rangle + t\langle 1, 4, -1 \rangle$

$= \langle t, 4+4t, 2-t \rangle$

(c) Set up an integral that represents the distance the particle travels from $t = 4$ to $t = 7$. [4]

$|\vec{r}\,'(t)| = \sqrt{\frac{1}{t^2} + 4(t+1)^2 + 1}$

$\therefore L = \text{arc length} = \int_4^7 \sqrt{\frac{1}{t^2} + 4(t+1)^2 + 1}\, dt$

2. Compute the line integral of $f(x, y) = 4xy$ over the line segment from $(1, -2)$ to $(3, 0)$. [8]

Parameterize the curve C:

$\vec{r}(t) = \langle 1,-2\rangle + t\langle 3-1, 0-(-2)\rangle = \langle 1+2t, -2+2t\rangle$, $0 \le t \le 1$

$\Rightarrow \vec{r}\,'(t) = \langle 2,2\rangle$ and $|\vec{r}\,'(t)| = \sqrt{8}$.

$$\therefore \int_C f(x,y)\,ds = \int_C 4xy\,ds$$

$$= \int_0^1 4(1+2t)(-2+2t)\sqrt{8}\,dt$$

$$= 8\sqrt{8}\int_0^1 (1+2t)(-1+t)\,dt$$

$$= 8\sqrt{8}\int_0^1 (-1-t+2t^2)\,dt$$

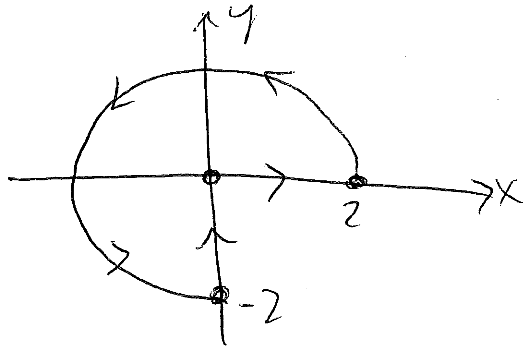
$$= 8\sqrt{8}\left(-t-\frac{1}{2}t^2+\frac{2}{3}t^3\right)\Big|_0^1$$

$$= 8\sqrt{8}\left(-1-\frac{1}{2}+\frac{2}{3}\right)$$

$$= 8\sqrt{8}\left(-\frac{5}{6}\right) = \frac{-20\sqrt{8}}{3}.$$

3. Let C be the curve given by the line segment from the origin to $(2,0)$, followed by the portion of the circle going counterclockwise from $(2,0)$ to $(0,-2)$, followed by the line segment from $(0,-2)$ back to the origin.

(a) Sketch the curve C. Be sure to label the direction of motion. [3]



C is closed.

(b) If $\vec{F} = \langle -2y, x\rangle$, evaluate the line integral of $\vec{F}$ over C. [4]

$$\int_C \vec{F}\cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_D (1-(-2))\, dA$$

$$= 3\iint_D 1\, dA = 3\cdot A(D) = 3\cdot\frac{3}{4}\pi\cdot 4 = 9\pi.$$

(c) If $\vec{F} = \langle x^2y, -xy^2\rangle$, evaluate the line integral of $\vec{F}$ over C. [4]

$$\int_C \vec{F}\cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_D (-y^2 - x^2)\, dA$$

$$= -\iint_D (x^2+y^2)\, dA = -\int_0^{\frac{3\pi}{2}}\int_0^2 r^2\cdot r\, dr\, d\theta$$

$$= \frac{-3\pi}{2}\int_0^2 r^3\, dr = \frac{-3\pi}{8} r^4\Big|_0^2 = -6\pi.$$

4. Let $\vec{F} = \langle 3x^2 - 2xy + 5, y^3 - x^2 \rangle$ be a vector field.

(a) Show that $\vec{F}$ is conservative. [4]

$$\frac{\partial Q}{\partial x} = -2x \ , \quad \frac{\partial P}{\partial y} = -2x$$

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \Rightarrow \vec{F} \text{ is conservative.}$$

(b) Determine a function $f(x, y)$ so that $\vec{F} = \vec{\nabla} f$. [4]

$$f(x,y) = \int (3x^2 - 2xy + 5)\, dx = x^3 - x^2 y + 5x + g(y)$$

$$\Rightarrow f_y = -x^2 + g'(y) = y^3 - x^2 \Rightarrow g'(y) = y^3$$

$$\Rightarrow g(y) = \frac{1}{4} y^4 + K$$

$$\therefore f(x,y) = x^3 - x^2 y + 5x + \frac{1}{4} y^4 + K$$

(c) If C is the circle $(x - 2)^2 + (y + 4)^2 = 9$ traversed clockwise, evaluate the line integral [4]

$$\int_C \vec{F} \cdot d\vec{r}$$

$\vec{F}$ is conservative and C is closed, so

$$\int_C \vec{F} \cdot d\vec{r} = 0.$$

5. Compute the work done by the force field $\vec{F} = \langle 2x, 3y^2, 4z^3 \rangle$ in moving a particle along the line segment from $(-2, 1, -1)$ to $(1, 1, 1)$ and then along a line segment back to the origin. [8]

Hint: What is the gradient vector for $f(x, y, z) = x^2 + y^3 + z^4$?

$$f(x,y,z) = x^2 + y^3 + z^4 \Rightarrow \vec{\nabla} f = \langle 2x, 3y^2, 4z^3 \rangle = \vec{F}$$

$$\Rightarrow \vec{F} \text{ is conservative}$$

Let C be the line segment from $(-2,1,-1)$ to $(1,1,1)$ followed by the line segment from $(1,1,1)$ back to $(0,0,0)$.

↳ C starts at $(-2,1,-1)$ and ends at $(0,0,0)$.

$$W = \int_C \vec{F} \cdot d\vec{r} = \int_C \vec{\nabla} f \cdot d\vec{r}$$

$$= f(0,0,0) - f(-2,1,-1)$$

$$= (0) - \left((-2)^2 + (1)^3 + (-1)^4 \right)$$

$$= -6.$$