University of Connecticut
Department of Mathematics

MATH 2110Q | PRACTICE EXAM 2 | FALL 2016

NAME: SOLUTIONS

DISCUSSION SECTION: ______

Read This First!

- Read the questions and instructions carefully.
- The available points for each problem are given in brackets.
- You must **show your work** to obtain full credit (and to possibly receive partial credit). Correct answers with no justification will not receive credit.
- Make sure your answers are clearly indicated, and cross out any work you do not want graded.
- Do not leave any blanks! Even if you do not arrive at an answer, show as much progress towards a solution as you can, and explain your reasoning.
- Calculators are allowed, but models that can do symbolic computations (TI-89 and above, including TI-NSpire) are not allowed.

Grading - For Administrative Use Only

Page:	1	2	3	4	5	Total
Points:	15	10	8	7	10	50
Score:						

1. Find and classify all critical points for the function $f(x,y) = \frac{1}{2}y^2 - \frac{1}{3}x^3 - xy + 2x + 5$ [8]

$f_x = -x^2 - y + 2 = 0 \Rightarrow y = 2 - x^2$

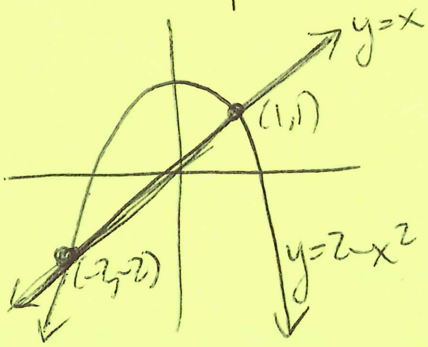
$f_y = y - x = 0 \Rightarrow y = x$

$\therefore y = x = 2 - x^2 \Rightarrow \underbrace{x^2 + x - 2}_{(x+2)(x-1)} = 0 \Rightarrow x = -2, 1$

$\therefore$ critical points are $(1,1)$, $(-2,-2)$

$f_{xx} = -2x$, $f_{xy} = -1$, $f_{yy} = 1$

$\Rightarrow D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = -2x - 1$

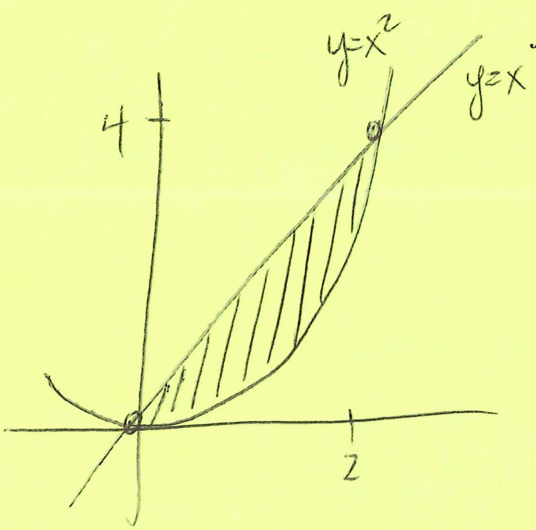


$D(1,1) = -3 < 0 \Rightarrow \boxed{(1,1) \text{ is a saddle}}$

$D(-2,-2) = 3 > 0$ and $f_{xx}(-2,-2) = 4 > 0$, so $\boxed{(-2,-2) \text{ is a local min}}$

2. Reverse the order of integration for the iterated integral [7]

$$\int_0^2 \int_{x^2}^{2x} f(x,y)\, dy\, dx.$$



$y = x^2 \Rightarrow x = \sqrt{y}$ $(x \geq 0)$

$y = 2x \Rightarrow x = \frac{1}{2}y$

So, $\int_0^2 \int_{x^2}^{2x} f(x,y)\, dy\, dx$

$= \boxed{\int_0^4 \int_{\frac{1}{2}y}^{\sqrt{y}} f(x,y)\, dx\, dy}$

3. Fill in the missing blanks in the following chart. [10]

Surface	Cartesian	Cylindrical	Spherical
Sphere	$x^2 + y^2 + z^2 = 16$	$r^2 + z^2 = 16$	$\rho = 4$
Cone	$z = \sqrt{x^2+y^2}$	$z = r$	$\phi = \frac{\pi}{4}$
Plane	$x = 3$	$r = 3\sec\theta$	$\rho \sin\phi \cos\theta = 3$
Paraboloid	$z = x^2+y^2$	$z = r^2$	(*) $\rho = \cot\phi \csc\phi$
Cylinder	$x^2+y^2 = 1$	$r = 1$	$\rho = \csc\phi$

(*) $z = x^2+y^2 \Rightarrow \rho\cos\phi = \rho^2\sin^2\phi$

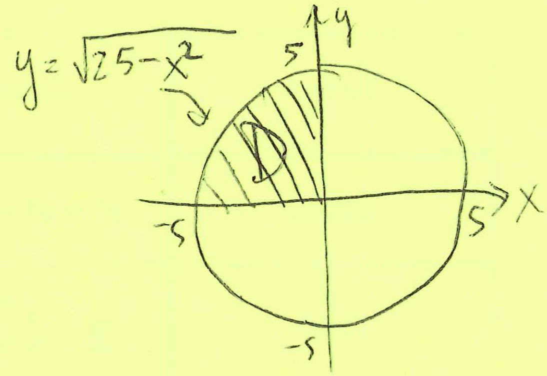
$\Rightarrow \cos\phi = \rho\sin^2\phi$

$\Rightarrow \rho = \dfrac{\cos\phi}{\sin^2\phi}$

$\Rightarrow \rho = \cot\phi\csc\phi$

4. Set up a triple integral that could be used to compute the volume contained between the xy-plane and the surface $z = 2(x^2 + y^2) + 3$ over the region D in the *second quadrant* enclosed by $x^2 + y^2 \leq 25$ using

(a) Cartesian coordinates. [4]



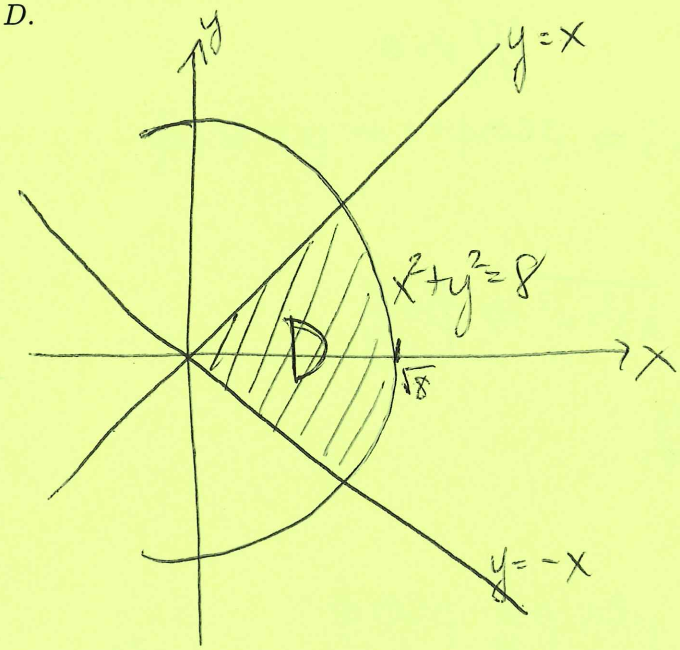
$$V = \iiint_E 1 \, dV$$

$$\boxed{= \int_{-5}^{0} \int_{0}^{\sqrt{25-x^2}} \int_{0}^{2(x^2+y^2)+3} 1 \, dz \, dy \, dx}$$

(b) Cylindrical coordinates. [4]

$$V = \boxed{\int_{\frac{\pi}{2}}^{\pi} \int_{0}^{5} \int_{0}^{2r^2+3} r \, dz \, dr \, d\theta}$$

5. Let D be the region in the xy-plane enclosed by $y = x$, $y = -x$, and $x^2 + y^2 = 8$, assuming $x \geq 0$. Sketch the region D and use a double integral in **polar coordinates** to compute the area of D. [7]



$$A = \iint_D 1\, dA = \boxed{\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{\sqrt{8}} r\, dr\, d\theta}$$

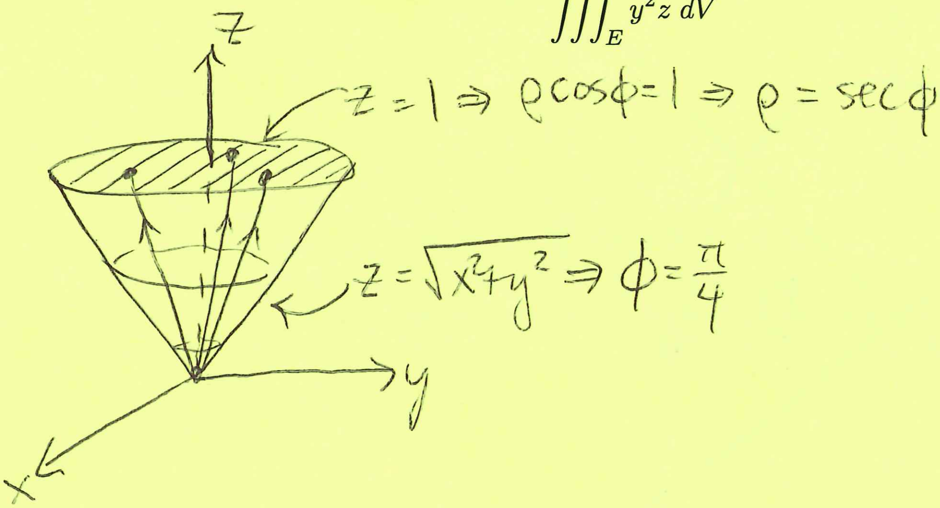
$$= \left(\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta\right)\left(\int_0^{\sqrt{8}} r\, dr\right)$$

$$= \frac{\pi}{2} \cdot \frac{1}{2} r^2 \Big|_0^{\sqrt{8}}$$

$$= \boxed{2\pi}$$

6. Write the following integral using **spherical coordinates** if the region E is bounded below by $z = \sqrt{x^2+y^2}$ and above by $z = 1$. **Do not evaluate**. [10]

$$\iiint_E y^2 z \, dV$$



$$\iiint_E y^2 z \, dV = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sec\phi} (\rho\sin\phi\sin\theta)^2(\rho\cos\phi)\rho^2\sin\phi \, d\rho\, d\phi\, d\theta$$

$$\boxed{= \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sec\phi} \rho^5 \sin^3\phi\cos\phi\sin^2\theta \, d\rho\, d\phi\, d\theta}$$