Name: ______________________________

Score: ________ /20

Partial Derivatives and Tangent Planes

Please staple your work and use this page as a cover page.

1. Find F_α and F_β for the function
$$F(\alpha,\beta) = \int_\alpha^\beta \sqrt{t^3+1}\, dt.$$

2. Given $f(x,y,z) = xy^2z^3 + \arcsin(x\sqrt{z})$, find f_{xzy}.

   Hint: Which order of differentiation is easiest?

3. In a study of frost penetration it was found that the temperature T at time t (measured in days) at a depth x (measured in feet) can be modeled by the function
$$T(x,t) = T_0 + T_1 e^{-\lambda x}\sin(\omega t - \lambda x),$$
   where ω and λ are positive constants.
   (a) Find $\partial T/\partial x$. What is its physical significance?
   (b) Find $\partial T/\partial t$. What is its physical significance?
   (c) Show that T satisfies the equation $T_t = kT_{xx}$ for some constant k.

   (The equation $T_t = kT_{xx}$ is a famous partial differential equation called the heat equation. Showing that this function $T(x,t)$ satisfies the equation means that we have found a solution.)

4. Find an equation of the tangent plane for $z = x\sin(x+y)$ at the point $(-1,1,0)$.

5. Show that each function on the left can be approximated by the function on the right near $(0,0)$.
   (a) $\dfrac{2x+3}{4y+1} \approx 3 + 2x - 12y$
   (b) $\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$

6. We don't know the equation that defines a certain surface S, but we are able to determine two tangent vectors at the point $(2,1,3)$ on the surface, namely
$$\vec{v}_1 = \langle 3, 0, -4\rangle \qquad \text{and} \qquad \vec{v}_2 = \langle 1, 6, 2\rangle.$$
   Using this information, determine an equation for the tangent plane at $(2,1,3)$.

7. Show that the function $f(x,y) = x^2 + 4y^2 - 4xy + 2$ has an infinite number of critical points and that $D = 0$ at each one. Use another method to show that every critical point is actually a local minimum.

   Hint: What type of shape are the x- and y-traces of this function's graph?

8. For a function of one variable, it is impossible for a continuous function to have, for example, two local maxima without a local minimum (or vice versa). However, for functions of two or more variables, such functions exist. Given the function
$$f(x,y) = -(x^2-1)^2 - (x^2y - x - 1)^2,$$
   show that $(-1,0,0)$ and $(1,2,0)$ are critical points and that both are local maxima. (In fact, it can be shown that these two points are the only critical points for this function).