Math 1070 Exam Final Exam Review

1. Bluth's Original Frozen Banana Stand sells two different banana treats. A fudge covered frozen banana with nuts and a double dipped frozen banana. The fudge banana is made with 20 ounces of chocolate and 12 ounces of nuts. The double dipped banana is made with 30 ounces of chocolate and 2 ounces of nuts. The banana stand has in stock 30,000 ounces of chocolate and 3,600 ounces of nuts. A profit of 2 dollars is made on the fudge banana and a profit of 3 dollars is made on the double dipped banana. As they say, "There is always money in the banana stand" so, how many of each banana should be made in order to maximize profits?

- Let x = fudge covered banana
  y = double dipped banana
- Want to maximize $P = 2x + 3y$ subject to:

Chocolate → $20x + 30y \leq 30{,}000$

Nuts → $12x + 2y \leq 3{,}600$

Graph the inequalities:

$20(0) + 30y = 30{,}000$ $\quad$ $20x + 30(0) = 30000$

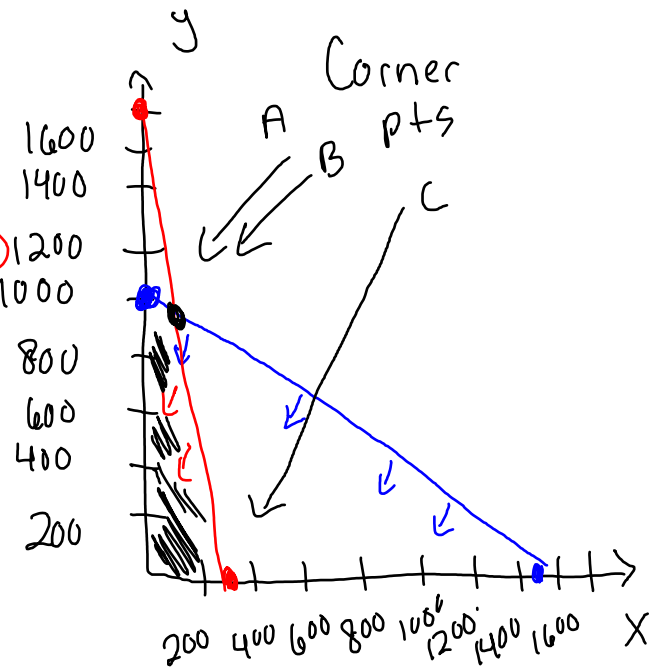
$y = 1000 \rightarrow (0, 1000)$ $\quad$ $x = 1500 \rightarrow (1500, 0)$

$12(0) + 2y = 3600$

$y = 1800 \rightarrow (0, 1800)$

$12x + 2(0) = 3600$

$x = 300 \Rightarrow (300, 0)$



A: $(0, 1000)$ $\quad$ B: Need to solve $\quad$ C: $(300, 0)$

$y = 1800 - 6x$

$20x + 30(1800 - 6x) = 30{,}000$

$20x + 54000 - 180x = 30{,}000$

$24000 = 160x$

$x = 150 \Rightarrow y = 900$

Profit maximized at B/A

$P = 2(150) + 3(900) = \$3{,}000$

2. Solve the following system of equations using Gauss-Jordan Elimination.

$x + y + z = 8$
$y - 12z = 0$
$x - 3z = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & 1 & -12 & 0 \\ 1 & 0 & -3 & 0 \end{array}\right] \quad R1 = R1 - R2$$

$$\begin{array}{cccc} 1 & 1 & 1 & 8 \\ -\ 0 & 1 & (-12) & 0 \\ \hline 1 & 0 & 13 & 8 = R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 13 & 8 \\ 0 & 1 & -12 & 0 \\ 1 & 0 & -3 & 0 \end{array}\right] \quad R3 = R3 - R1$$

$$\begin{array}{cccc} 1 & 0 & -3 & 0 \\ -\ 1 & 0 & 13 & 8 \\ \hline 0 & 0 & -16 & -8 = R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 13 & 8 \\ 0 & 1 & -12 & 0 \\ 0 & 0 & -16 & -8 \end{array}\right] \quad R1 = 16R1 + 13R3$$

$$\begin{array}{cccc} 16 & 0 & 208 & 128 \\ +\ 0 & 0 & -208 & -128 \\ \hline 16 & 0 & 0 & 0 = R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 16 & 0 & 0 & 0 \\ 0 & 1 & -12 & 0 \\ 0 & 0 & -16 & \end{array}\right] \quad R2 = 16(R2) + -12(R3)$$

$$\begin{array}{cccc} 0 & 16 & -192 & 0 \\ +\ 0 & 0 & 192 & 96 \\ \hline 0 & 16 & 0 & 96 = R2 \end{array}$$

$$\left[\begin{array}{cc|} 16 & 0 \\ 0 & 16 \\ 0 & 0 \end{array}\right] \quad R3 -$$

3. You are being dealt five cards from a standard deck of 52 playing cards. Find the probability that you are dealt two clubs and three red cards.

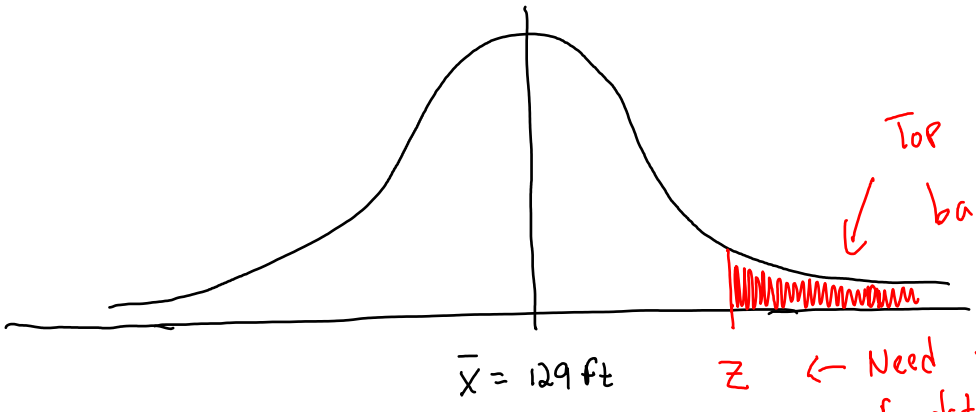
Sample space: $\binom{52}{5} = {}_{52}C_5 = 2,598,960$

Event space: $\binom{13}{2} \cdot \binom{26}{3} = {}_{13}C_2 \cdot {}_{26}C_3$

$= 202,800$

$P(\text{2 clubs and 3 red}) = \frac{202800}{2,598,960} \approx 0.078$

4. A researcher tests the braking distances of several cars. The braking distance from 60 miles per hour to a complete stop on dry pavement is measured in feet. The braking distances of a sample of cars are normally distributed, with a mean of 129 feet and a standard deviation of 5.18 feet. The company does not want to sell the top 10% of cars with the longest braking distances, what should the cutoff be?



$z = 1.28$ has 90% below!

$$X = z\sigma + \mu \quad \text{or} \quad X = zS + \bar{x}$$

$$x = 1.28(5.18) + 129$$

$$x = 135.63 \text{ ft}$$

Cars w/ a braking distance of 135.63 ft or more should not be sold!

5. Charter Oak Credit Union has a two year CD at 0.5% interest compounded monthly. You want to have $1100 saved at the end of the two years, how much do you have to deposit?

$P = ?$

$r = 0.005$

$t = 2$

$m = 12$

$F = 1100$

$$1100 = P\left(1 + \frac{0.005}{12}\right)^{12(2)}$$

6. A firm manufactures tables and desks. To produce each table requires 1 hour of labor, 10 square feet of wood, and 2 quarts of finish. To produce each desk requires 3 hours of labor, 20 square feet of wood, and 1 quart of finish. Available is at most 45 hours of labor, at more 350 square feet of wood, and at most 55 quarts of finish.

a) Write a system of inequalities and graph the feasible region.

b) Find all of the corner points.

c) The tables and desks yield profits of $4 and $3 each, respectively. Find the number of each product to be made is order to maximize profit.

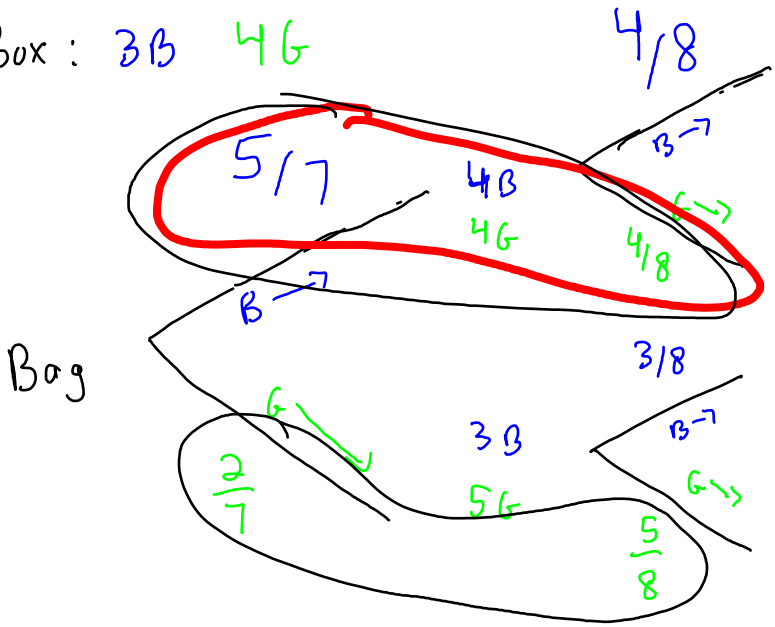
Not a great question

Do #1!

7. A bag contains five blue and two green jelly beans. A box contains three blue and four green jelly beans. A jelly bean is selected at random from the bag and is placed in the box. Then a jelly bean is selected at random from the box. If a green jelly bean is selected from the box, what is the probability that the transferred jelly bean was blue?

Bag: 5B 2G

Box: 3B 4G



$P(\text{Blue trans} \mid \text{green selected}) =$

$$\frac{\frac{5}{7}\left(\frac{4}{8}\right)}{\frac{5}{7}\cdot\frac{4}{8}+\frac{2}{7}\cdot\frac{5}{8}}$$

$$\approx \frac{2}{3}$$

8. In how many ways can 5 members of a basketball team line up to take a picture?

Fudamental Counting Principle

$5! =$ 120 ways to line up

9. Callahan Auto Parts has hired 12 new employees, and must assign 8 to the day shift and 4 to the night shift. Assume that the 12 employees consist of 6 men and 6 women and that the assignments to day and night shift are made at random.

(a) What is the probability that all 4 of the night-shift employees are men?

(b) What is the probability that at least one of the night-shift employees is a woman?

(a) $P(\text{all men}) = \dfrac{\binom{6}{4}}{\binom{12}{4}} = \dfrac{15}{495} \approx 0.\overline{03}$

(b) $P(\text{at least on woman}) = 1 - P(\text{all men})$

$= 1 - 0.\overline{03}$

$= 0.\overline{96}$

10. Find the amount needed to deposit into an account today that will yield a typical pension payment of $25,000 at the end of each of the next 20 years at the annual interest rate of 7%.

$PMT = 25,000$

$m = 1$ $\qquad i = \frac{r}{m} = \frac{0.07}{1} = 0.07$

$t = 20$

$r = 0.07$ $\qquad n = mt$ $\qquad n = 20$

Solve for P.V.

$$P.V. = 25,000\left(\frac{1-(1+0.07)^{-20}}{0.07}\right)$$

$\approx \$264,850.36$