Q-Center Math 1070 Exam #2 Review

November 8, 2016

#1 Arsenic is a compound that occurs naturally in very low concentrations. Arsenic blood concentrations in healthy adults are normally distributed with mean $\mu = 3.2$ micrograms per deciliter (*mg/dl*) and a standard deviation of $\sigma = 1.5$ *mg/dl*.

(a) A healthy adult is randomly seleceted, find the probability that the adult has between 2 and 4 *mg/dl* of arsenic in their blood.

Here we want to find the area under the normal curve between $x = 2$ and $x = 4$. This implies we must covert the x values to z-scores and then use the normal table. That is;

$$z_2 = \frac{2 - 3.2}{1.5} = -0.8 \text{ and } z_4 = \frac{4 - 3.2}{1.5} = 0.53$$

so we want

$$P(-0.8 \leq z \leq 0.53) = P(z \leq 0.53) - P(z \leq -0.8) = 0.7019 - 0.2119 = 0.49$$

Therfore, approximately 49% of healthy adults have between 2 and 4 *mg/dl* of arsenic in their blood.

(b) Find the top 5% of arsenic concentrations in healthy adults.

This is asking us to find the arsenic concentrations that cuts off the highest 5% of concentration levels of all healthy adults. So, top 5% implies we want a z-score with 95% of the area under the normal curve below it. We must do a revese look up of the normal table, find an area closest to 0.95 and then trace back to a z-score. However, we see that 0.95 is exactly in the middle of 0.945 and 0.9505; therefore we can use $z = 1.645$ as the appropriate z-score with 95% area below. Now we simply solve for x in the z-score formula.

$$1.645 = \frac{x - 3.2}{1.5}$$

Thus we see that $x = 5.6675$ which tells us that an arsenic contration of 5.6675 *mg/dl* or above is in the top 5% of concentrations in healthy adults.

#2 The 2014 New England Patriots had an amazing year ending with a Super Bowl win. Lets compare how the Patriots offense performed on the road versus at home. Below; are the number of points scored by the offense on the road and at home:

Road: $20,\ 30,\ 14,\ 37,\ 42,\ 21,\ 23,\ 17$

Home: $16,\ 43,\ 27,\ 51,\ 43,\ 34,\ 41,\ 9$

(a) Compute the mean, variance, and standard deviation for the points scored at home (do this by hand).

$$\mu = \frac{16 + 43 + 27 + 51 + 43 + 34 + 41 + 9}{8} = 33$$

So the patriots scored on average 33 points per game at home. Now to compute the variance and standard deviation we have;

$$\sigma^2 = \frac{(16 - 33)^2 + (43 - 33)^2 + (27 - 33)^2 + \cdots + (9 - 33)^2}{8} \approx 186.25$$

and

$$\sigma = \sqrt{\sigma^2} = \sqrt{186.25} \approx 13.65, \text{ points}$$

(b) Suppose the mean and standard deviation for the Patriots offense on the road was 25.5 points and 9.26 points respectively. Using this information and your answers from part (a), did the Patriots perform better on the road or at home and were the Patriots more consistent on the road or at home? Explain.

The Patriots offense performed better at home as compared to on the road since their mean points per game scored was 7.5 more (about a touchdown and PAT more per game). However, the offense has more consistent scoring on the road since the standard deviation was lower with 9.26 points.

#3 (a) You are trying to save for a new car and decide to deposit $\$10,000$ into a 2 year CD at Charter Oak Credit Union earning 1.48% compounded quarterly. How much money will you have saved after the 2 years?

Here we must use the formula $F = P(1 + \frac{r}{m})^{mt}$ where F is our unknown, $P = 10,000,\ t = 2,\ r = 0.0148,$ and $m = 4$. Therefore our future value on the CD is,

$$F = 10,000\Big(1 + \frac{0.0148}{4}\Big)^{2(4)} = \$10,299.86$$

(b) Repeat the question with daily compounded interest and compare.

Here we need only to change $m = 4$ to $m = 365$ and we get that the future value of the CD is $F = \$10,300.42$ giving 56 cents more over the 2 years; WOW!!!!!!

#4 Suppose you deposit a fixed amount of money every month for 5 years into an account with an annual interest rate of 2.75% compounded monthly. After 5 years, you stop depositing money and let the money in the account gain interest for the next 5 years, still at an annual rate of 2.75% compounded monthly. If the total amount in the account at the end of the 10 years is $\$7369.98$ what were the payment amounts to be made during the first five years?

First notice that this problem involves both depositing a fixed amount monthly as well as compound interest, therefore we will need the following formulas.

$$F.V. = PMT\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}} \text{ and } F = P\left(1+\frac{r}{m}\right)^{mt}$$

Now, the key with this question is to think of the two accounts on a timeline over 10 years with something happening at the end of the 5^{th} year (or in the middle). For the first 5 years we are depositing a fixed amount each month. This implies we are working with an annuity, but, of course we do not know what the fixed payment (PMT) is which then implies that we do not know the future value $(F.V.)$. For the last 5 years the money from the annuity sits in a compound interest bank account for which we only know the future value (F) of, i.e. $F = \$7369.98$. This implies that we can solve for the principal (P) on the compounded account; but what is the principal with respect to this problem?? Well, this account starts at the end of the first five years with money from the annuity, so, we have that the futre value of the annuity gives the principal for the compounded account, i.e. $F.V. = P$. With a little bit of manipulation we get,

$$P = \frac{7369.98}{(1+\frac{0.0275}{12})^{12(5)}} = PMT\frac{(1+\frac{0.0275}{12})^{12(5)}-1}{\frac{0.0275}{12}}$$

Carefully simplifying the left-hand and right-hand side we get,

$$6424.201252 = PMT(64.24199) \implies PMT = \$100.00$$

#5 Solve the following system of equations using your choice of method:

$$-5x + y = -3 \qquad 3x - 8y = 24$$

A quick analysis of the equations suggest solving by substitution will be the easiest. So,

$$-5x + y = -3 \implies y = -3 + 5x$$

Substituting into the second equations gives,

$$\begin{aligned} 3x - 8(-3 + 5x) &= 24 \\ 3x + 24 - 40x &= 24 \\ -37x &= 0 \\ x &= 0 \end{aligned}$$

Substituting $x = 0$ into either of the two equations gives $y = -3$.

#6 Isiah Thomas from the Boston Celtics as of November 7 in the 2016-2017 season has made 82.7% of his free throws. Let X be the number of free throws made.

(a) Suppose Isiah attempts 5 free throws in a game, find the probability he makes exactly 4 of them.

This is a Bernoulli Trial with $n = 5,\ p = 0.827$, and $q = 1 - p = 0.173$. Therefore,

$$P(X = 4) = C(5, 4)(0.827)^4(0.173)^1 \approx 0.4046$$

(b) Construct a probability distribution for X.

Using the same forumla above, just changing the number of successes and failures for each X value we get.

X	0	1	2	3	4	5
$P(X)$	0.0002	0.0037	0.0354	0.1693	0.4046	0.3869

Notice that $\sum_{n=0}^{5} P(X_n) \approx 1$. This is a good way to check if your calculations are correct (or at least appear correct).

(c) Find the expected number of free throws made.

We have $E(X) = \sum_{n=0}^{5} (X_n)(P(X_n))$. Therefore we need,

$$E(X) = 0(0.0002) + 1(0.0037) + 2(0.0354) + 3(0.1693) + 4(0.4046) + 5(0.3869) = 4.1353$$

This implies that when making 5 free throws during a game Isiah will make on average 4.1353 of the 5 free throws.

#7 Winston McCall is purchasing a home for $\$250,000$. Charter Oak Credit Union has a 15 year fixed rate loan at 2.875%. The bank is requiring a 15% down payment. Find the down payment for the mortgage, the monthly payments, the total amount to be paid back, as well as the amount of interest to be paid.

In order to find the down payment to the bank we simply find 15% of the house's selling price. That is,

$$250,000(0.15) = \$37,500$$

Now, to find the payment for the mortgage we first note that we are only borrowing $\$212,500$ since the $\$37,500$ is already paid to the bank. Therefore, rearranging the present value formula for an annuity we see,

$$PMT = \frac{212,500\left(\frac{0.02875}{12}\right)}{\left(1-\left(1+\frac{0.02875}{12}\right)^{-12(15)}\right)} = \$1,454.75$$

Now the total amount to be paid back is found by multiplying the monthly payment by how many times it needs to be paid, that is,

$$1,454.75(12)(15) = \$261.855$$

Finally, the interest paid is the total amount paid back minus how much is borrowed, thus we get

$$261,855 - 212,500 = \$49,355$$

#8 A baker will supply 16 jumbo cinnamon rolls to a cafe at a price of $\$1.70$ each. If she is only offered $\$1.50$, then she will supply 4 fewer rolls to the cafe. The cafe's demand for jumbo cinnamon rolls is given by $p = D(x) = -0.16x + 7.2$.

Find the supply equation $p = S(x)$.

First we must be careful and note that the number of cinnamon rolls is dependent on the price. Therefore,

$$slope = m = \frac{1.7 - 1.5}{16 - 12} = 0.05$$

Then,

$$y - 1.7 = 0.05(x - 16) \implies y = 0.05x + 0.9 \implies p = S(x) = 0.05x + 0.9$$

Now, in order to find the equilibrium price and quantity we must first set $S(x) = D(x)$ and solve for the quantity x. So,

$$0.05x + 0.9 = -0.16x + 7.2$$

$$0.21x = 6.3$$

$$x = 30 \text{ buns}$$

Substituting the quantity back into either equation gives a price of $\$2.40$.